SHORT RULE ARITHMETIC AND ACCOUNTANT'S REFEREE

· DAVID ·

THE

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ILLUSTRATED

Comprising an original treatise on Addition, Subtraction, Multiplication and Division, showing how these processes may be shortened when applied to the practical computations of business, together with easy, short methods in Fractions, Decimals, Interest, and many rapid rules in buying and selling the various articles of commerce, to which is added rules for finding the day of the week corresponding to any date forever, Easter Sunday, etc.,

TOGETHER WITH

A large but ingeniously condensed collection of simple rules for all kinds of measurements.

THE WHOLE DESIGNED AS

A SELF INSTRUCTOR

For home study, a Guide and Reference for the accountant and practical man in the office and workshop, and as a text-book for special classes in Schools and Business Colleges.

BY

WM. K. DAVID.

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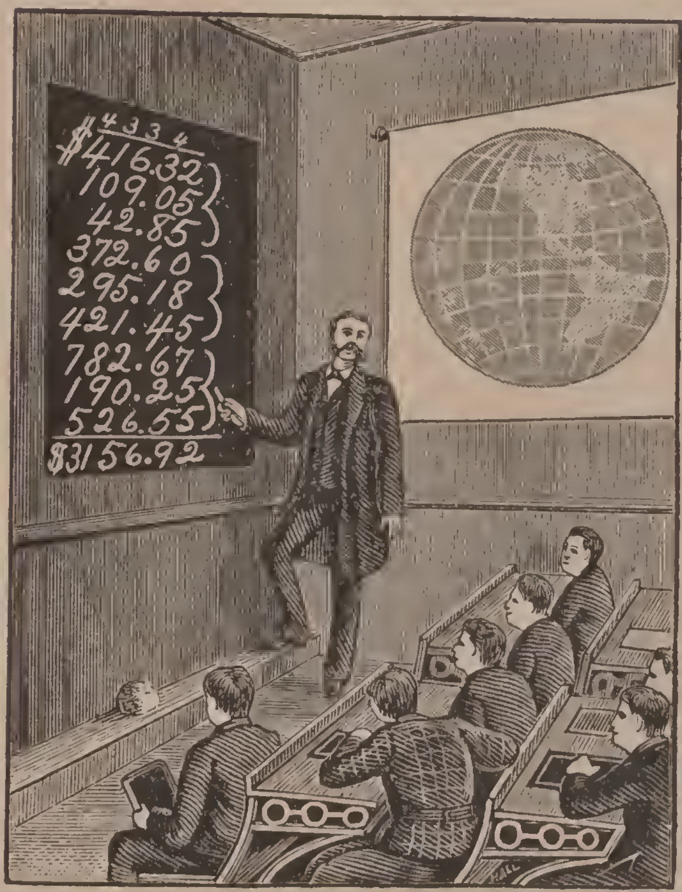
PREFACE.

This book is designed for thinkers, young and old, who have acquired the rudiments of numbers in any ordinary arithmetic. It arrangement is convenient for special class teaching in schools and business colleges, or to be used by teacher or student in connection with the numerous arithmetical text books. It is especially designed as a self-instructor and referee for the practical accountant and user of figures in every line of work in this age of quick thought and rivalry in business. The methods and forms are short, simple, easily comprehended, and correct. The majority of the rules and tables are original, and cannot be found in any other work. To those who are familiar with the contents of School Arithmetics, Ready Reckoners, Calculators, etc., this will be apparent at a glance.

THE AUTHOR.

Yours truly,
Wm. K. David.

BUSINESS ADDITION.



GROUPING METHOD.

To add **quickly** and **correctly** is an accomplishment that can be attained by any person who will diligently follow our instructions. Thousands of pages have been written on the subject of **rapid** or **lightning** addition. Some authors advocate methods of adding two or more columns at

once, but experience has proven that but few can afford to spend their time in long hours of practice necessary to master it; and others have taught what is commonly known as the word system, requiring many pages of explanation, which after all is simply a system of grouping.

The author has fully demonstrated that by adding a **single** column at a time the student can learn to grasp the sum of at least **three** figures at a **glance,** and by uniting the result to the sum of other groups as they are seen, an example can be added with **greater ease** and **speed** than by any other system.

This of course requires practice, not only in perceiving the sum of each group quickly, but in adding the groups. The first thing to learn is to name the sum of any three digits as quickly as to pronounce a syllable of three letters. We choose the combination of three figures in preference to combinations of twos, because by a little practice the sum of three figures can be seen in an instant, and a great saving of time is made in adding the groups, as there are fewer of them and they are just as easily added as smaller groups. The beauty of learning to group figures in this manner will become apparent after the student is thoroughly familiar with groups of threes. Frequently the eye will intuitively pass on beyond three figures, and in an instant—like pronouncing a long word—perceive the sum of six, seven, eight, nine, or more figures at a glance and at the expenditure, seemingly, of no mental effort.

The following table is arranged for practice and drill, and shows all the possible combinations of the nine digits in groups of three. It is not necessary to memorize the groups, but familiarize yourself with them until you can easily read their sums. This can be done by glancing over the table at random, or by writing on paper or slate any number of columns three deep and adding them as quickly as possible. When this is acquired practice adding the combinations together by taking longer columns of figures:

GROUPING ADDITION TABLE

Containing all combinations of the nine digits in groups of three. It will be observed that no group can add more than 27 nor less than 3:

27	9 9 9	1 4 9, 1 5 8, 1 6 7, 2 3 9, 2 4 8, 2 5 7, 2 6 6, 3 3 8, 3 4 7, 3 5 6, 4 4 6, 4 5 5	14
26	8 9 9	1 3 9, 1 4 8, 1 5 7, 1 6 6, 2 2 9, 2 3 8, 2 4 7, 2 5 6, 3 3 7, 3 4 6, 3 5 5, 4 4 5	13
25	8 8 9, 7 9 9	1 2 9, 1 3 8, 1 4 7, 1 5 6, 2 2 8, 2 3 7, 2 4 6, 2 5 5, 3 3 6, 3 4 5, 4 4 4	12
24	8 8 8, 7 8 9, 6 9 9	1 1 9, 1 2 8, 1 3 7, 1 4 6, 1 5 5, 2 2 7, 2 3 6, 2 4 5, 3 3 5, 3 4 4	11
23	7 8 8, 7 7 9, 6 8 9, 5 9 9	1 1 8, 1 2 7, 1 3 6, 1 4 5, 2 2 6, 2 3 5, 2 4 4, 3 3 4	10
22	7 7 8, 6 8 8, 6 7 9, 5 8 9, 4 9 9	1 1 7, 1 2 6, 1 3 5, 1 4 4, 2 2 5, 2 3 4, 3 3 3	9
21	7 7 7, 6 7 8, 6 6 9, 5 8 8, 5 7 9, 4 8 9, 3 9 9	1 1 6, 1 2 5, 1 3 4, 2 2 4, 2 3 3	8
20	2 9 9, 3 8 9, 4 7 9, 4 8 8, 5 6 9, 5 7 8, 6 6 8, 6 7 7	1 1 5, 1 2 4, 1 3 3, 2 2 3	7
19	1 9 9, 2 8 9, 3 7 9, 3 8 8, 4 6 9, 4 7 8, 5 5 9, 5 6 8, 5 7 7, 6 6 7	1 2 3, 1 1 4, 2 2 2	6
18	1 8 9, 2 7 9, 2 8 8, 3 7 8, 3 6 9, 4 5 9, 4 6 8, 4 7 7, 5 5 8, 5 6 7, 6 6 6	1 1 3, 1 2 2	5
17	1 7 9, 1 8 8, 2 6 9, 2 7 8, 3 5 9, 3 6 8, 3 7 7, 4 4 9, 4 5 8, 4 6 7, 5 5 7, 5 6 6	1 1 2	4
16	1 6 9, 1 7 8, 2 5 9, 2 6 8, 2 7 7, 3 4 9, 3 5 8, 3 6 7, 4 4 8, 4 5 7, 4 6 6, 5 5 6	1 1 1	3
15	1 5 9, 1 6 8, 1 7 7, 2 4 9, 2 5 8, 2 6 7, 3 3 9, 3 4 8, 3 5 7, 3 6 6, 4 4 7, 4 5 6, 5 5 5		

The following example will explain the process, together with an unique method of disposing of the carrying figures:

$$
\begin{array}{r}
\scriptstyle 8\quad 7\quad\ \ \\
\left.\begin{array}{r}913\\345\\852\end{array}\right\}\\
\left.\begin{array}{r}34\\946\\362\end{array}\right\}\\
\left.\begin{array}{r}753\\191\\244\end{array}\right\}\\
\left.\begin{array}{r}967\\505\\638\end{array}\right\}\\
\left.\begin{array}{r}776\\242\\452\end{array}\right\}\\
\left.\begin{array}{r}665\\524\\122\end{array}\right\}\\
\hline
9531\quad
\end{array}
$$

Commencing at the bottom of the right-hand column we find the sum of the first group is 11, which, added to the sum of the next group (10),=21. Proceed in like manner with the remaining groups, adding each new group to the sum of the groups already found, until the complete sum (71) of the column is obtained. Write the ^{7} (carrying figure) over the next column and the unit figure (1) under the column added. It will be observed that we can now leave our work, and upon resuming our addition the carrying figure being in its proper place we add it to the sum of its column, which we find to be 83, as follows:

$$10+16+9+18+13+10+{}^{7}=83.$$

Write the ^{8} above the next column and 3 below the column added. The sum of the next column, found as before (including carrying figure), is 95, which place in the answer and the sum is complete.

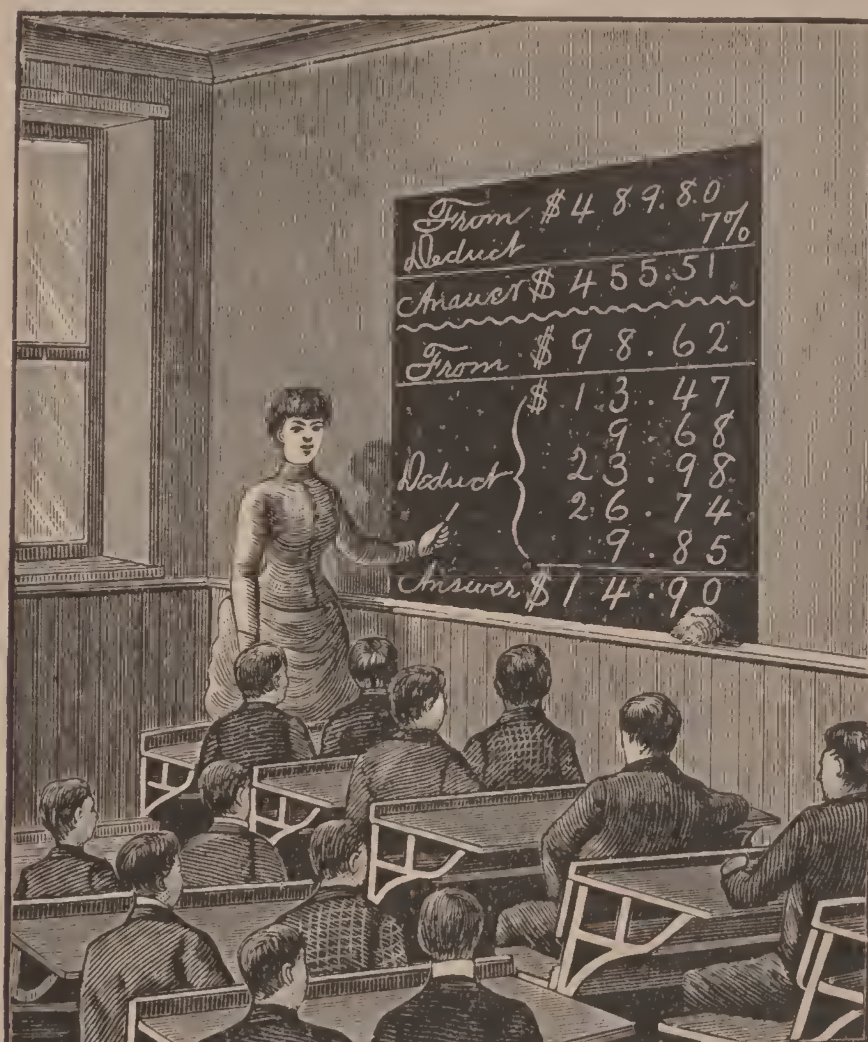
To obviate the possibility of mistaking the carrying figures for an entry, a light line may be drawn above the columns and the carrying figures written above it slightly smaller than the others, in red ink or with a lead pencil. If the sum to carry contains two figures write them over the proper column as in the case of one figure.

PRACTICAL PROOF OF ADDITION.

Writing the carrying figures in this manner affords a neat and decidedly accurate proof of addition, as follows: Commence with the carrying figure of the **left-hand** column and add **downward.** If the result equals the figures found below the column in the answer (95) we can fairly conclude that thus far we are correct. Begin with the carrying figure of the next column and we find the sum is 83, which corresponds to the ^{8} over the preceding column and the 3 in the answer. The sum of the last column added downward is 71'

which corresponds to the [7] over the preceding column and the 1 in the answer. A very little practice will enable any one to add upward or downward with equal facility.

SUBTRACTION.



SHORT BUSINESS METHODS.

There are many cases where substraction may be more easily performed by a process of **adding.** In subtraction of simple numbers it does not surpass the ordinary method to any great extent, although many accountants who are familiar with the process prefer it. In obviating the

tedious subtractions in long division (see division), and also in many special cases met with quite often in actual business, the principle can be applied to decided advantage.

We will first explain the method as applied to simple numbers:

From	**7865**
Deduct	**5794**
	2071

EXPLANATION.—We begin at the right, as in the ordinary rule. Our object is to find the figure which when **added** to the 4 will make it **equal** the figure **above** it. This figure is **1**, which we write in the answer. When the figure **below** is **larger** than the corresponding figure **above** we must find a figure which when **added** to the **lower figure** will make the **unit figure** of their **sum** equal the **upper figure**, as follows: We see that 9 is greater than the 6 above it, therefore we must make 9 equal 16. To do this requires a **7**. Write the **7** in the answer and carry the 1 to the next lower figure (7) we get 8; 8 lacks **0** to make it equal the 8 above it, so write **0** in the answer. We next perceivethat **2 added** to 5 will make it equal the 7 above it, and by writing the **2** in the answer beneath we have the complete remainder.

EXAMPLES FOR PRACTICE.

From	9872	*From*	87596	*From*	65283
Deduct	6581	*Deduct*	79416	*Deduct*	47164
	3291		8180		18119

BOOK-KEEPERS' SUBTRACTION.

Frequently several numbers or entries are to be taken from a given amount; as, for instance, a firm sells a bill of goods amounting to $827.56 on which there were several payments, as follows: $125.45, $236.50, $84.75, $72.25. How much remains unpaid?

Ordinary Method.		Short Rule.
		$827.56
$125.45		**125.45**
236.50		**236.50**
84.75	**$827.56**	**84.75**
72.25	**518.95**	**72.25**
$518.95	**$308.61**	**$308.61**

EXPLANATION.—The sum of the first right-hand column of payments is 15. The figure necessary to make the unit figure of this sum equal the corresponding figure (6) of the subtrahend is **1**, i. e., **1 added** to 15=16; we write the **1** in the answer and carry 1 (tens) to the sum of the next column of payments, which gives 19. As the 9 in 19 is greater than the 5 in the subtrahend we find it will take **6** to make the unit figure equal it; 19+**6**=25, so we write the **6** in the answer and carry 2 to the sum of the next column of payments, which gives 19; **8**+19=27, making the unit figure equal the corresponding figure of the subtrahend. Writing down the **8** and carrying 2 to the sum of the next column we get 22. 22 requires **0** to make its unit figure equal the 2 above, so we write down **0** and carry 2 to the sum of the next column, which gives 5. We find that **3 added** to 5 will equal the 8 of the subtrahend, which, when written below, completes the remainder. While this requires considerable trouble to explain properly, it will be found very easy in practice.

DISCOUNT SUBTRACTION.

This principle affords a decidedly neat and very easy short rule for deducting a required percentage from any number. For instance, a wholesale house sells a bill of goods amounting to $762.00 allowing a discount of 8% for cash. What is the net amount:

Short Rule.	Ordinary Method.
$762.00	**$762.00**
.08	**.08**
$701.04	**$60.9600**
	$762.00
	60.96
	$701.04

EXPLANATION.—We simply multiply 762 by 8 and write in the answer the figures which when **added** to the product will equal $762.00, as follows: 8×2=16. Requiring a 4 to make 20 (next higher order of tens, as the figure on the right is 0,) we write **4** in the answer and carry the 2. 8×6=48, to which we add the 2, which gives 50. As its unit figure is 0, which corresponds to the 0 above, we write down **0** in the answer and carry 5 to the product of 8×7, which gives 61. 1 added to 61 will equal the 62 above, so we write down **1** and a **0** under the 6, and bringing down the remaining 7 we get the complete remainder. When cents occur in the amount multiply your per cent by the cents and reserve the

tens figure of your product as a carrying figure to add to your product of the per cent by the first dollar figure, as follows:

We perceive that 6 (%) multiplied by the 5 (dimes figure) equals 30; disregarding the 0 we carry 3 to the product of 6×6, which gives 39. Then, proceeding as above, we always get the answer to the nearest cent.

From	$256.50
Deduct	6 %
	$241.11

The preceding rule is preferable to any other where the per cent of discount can be represented by a **single** figure, but where the per cent is represented by **two** or **more** figures, as 30%, 45%, etc., it is best to proceed as follows:

From a bill of $926.75 deduct 40%. What is the net bill?

EXPLANATION.—The ordinary method is to find 40% of the amount of the bill and then subtract the percentage thus found from it. We can obviate this long subtraction by taking the difference of the rate per cent of discount and 100%, which can be easily done mentally, and multiply this difference by the amount of the bill. In this example the discount is 40%, which lacks 60% of 100%, or 60 cents of each dollar remains to be paid. We therefore simply multiply the number of dollars by .60. $926.75×.60=$556.05, answer. If the discount had been 30% we would have multiplied by .70, if 50% by .50, if 60% by .40, if 65% by .35, etc

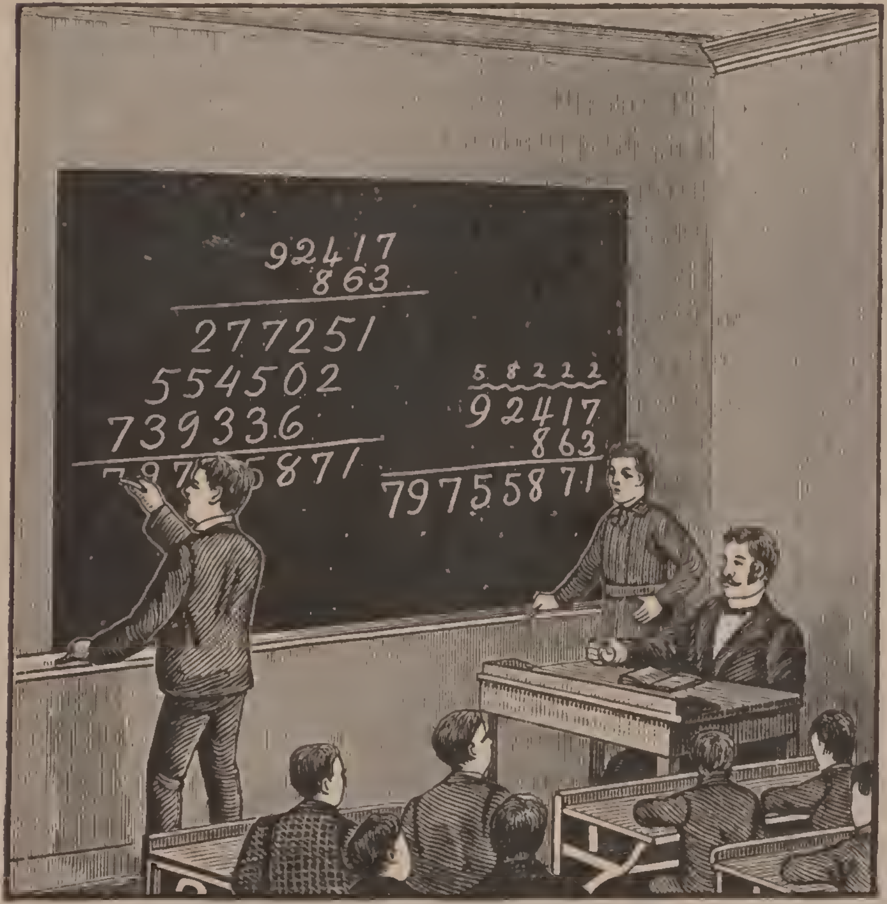
Frequently a series of rates per cent of discount are adopted by jobbers and manufacturers, as for instance, 30%, 20%, and 10%. The object in this case is not to give a discount of 60% **straight**, but to first deduct 30% from the **list** price and 20% from the remainder and 10% from the remainder thus obtained which will give the **net** price of their wares.

A jobber sells a bill of hardware amounting to $400, at a discount of 40%, 30%, and 10%.

EXPLANATION.—This may be performed in two ways quite different from the ordinary method and much easier. First, we can deduct 40% from 100%, which gives 60%; $400×.60=$240 (first remainder). We now deduct the next per cent in the series (30%) from 100%, which gives 70%; $240×.70=$168 (second remainder). The last per cent in the series being 10 we deduct it from 100%, which gives 90%; $168×.90=$151.20, net amount of bill. Another method is to subtract each rate per cent of the series separately from 100%, and by multiplying all the remainders together we get the **net** per cent to be paid. In the above example we would take the differences of the per cents and 100% and multiply them together, as follows: .60×.70×.90=.378 or $37\frac{8}{10}$% of the list price to be paid; $400×.378=$151.20, answer same as above. 100%—$37\frac{8}{10}$%=$62\frac{2}{10}$%, which is the net per cent discount of the series. The **net** discount of

any series can be found in this manner. Commercial salesmen and accountants can make out a net list of the series likely to be used, and their labor will be greatly facilitated. It is immaterial which per cent of the series is used first, 60 and 5 off being the same as 5 and 60 off, etc.

RAPID MULTIPLICATION.



PRACTICAL GENERAL METHODS.

The method known as **cross multiplication**, by which the partial products are added mentally, has been known by mathmeticians for ages, and while the rule is used practically by nearly every really expert accountant, it has received comparatively no attention from teachers, who do

not realize the importance of saving time in large commercial establishments. The reason of this neglect on their part can also be accounted for by the fact of the rule requiring more mental work than the ordinary method, and being more perplexing especially where the multiplier and multiplicand each consists of several figures. Many authors of arithmetics recommend the method to be taught in the schools in cases where the figures of the multiplier do not exceed two or three places, but further than that they are silent, and present no general remedy except to return to the ordinary rule. We will speak of the rule further on, together with some valuable improvements in connection with it. We will first show an original and easy method of multiplying which is quite general in its application and saves writing many figures:

Short Method.

Multiply	295
By	73
	88
	21535

Ordinary Method.

295
73
885
2065
21535

Explanation.—We draw two lines below the example with space enough between them in which to write as many lines of figures **less one** as there are significant figures in the multiplier. In this example, as there are two figures in the multiplier, we draw two lines below with space enough between them to write one line of figures. We first say 3 times 5 are 15. Write the 5 below the lower line and carry the 1 to the product of 3 times 9, which gives 28. Write 8 between the lines, one place to the left (or always under the figure of the multiplicand being used), and carry 2 to the product of 3 times 2, which gives 8. Write the sum between the lines to the left. Next we simply multiply 295 by the 7 (tens) as in ordinary multiplication, taking care to add the figures between the lines in their proper order, as follows: 7 times 5 are 35+8 (between the lines), which gives 43. Write 3 in the answer and carry 4 to the product of 7 times 9, which gives 67, to which we add the next figure (8) between the lines, making 75. Writing down 5 and carrying 7 to the product of 7 times 2 we get 21, which when written in the answer completes the product. The figures between the lines can be written above

the example, as in the illustration, without drawing an additional line below, but it is preferable, especially in larger numbers, to write the extra figures between the lines.

Short Method.		**Ordinary Method.**
Multiply	3476	3476
By	512	512
	695	6952
	348	3476
		17380
	1779712	1779712

EXPLANATION.—As we have three figures in our multiplier we leave a space between the lines sufficient to write **two** lines of figures. We first multiply 3476 by 2, which gives 6952. Write the unit figure (2) of the product **below** the lines and the remaining figures one place to the left **between** the lines. We next multiply 3476 by 1, which gives 3476, the unit figure of which we do not write, but add it to the first figure (5) between the lines, as follows: 1×6=6+5 (first figure between the lines)= 11. Write down the 1 below, and carry 1 to the product of 1×347, which gives 348. Write it one place to the left of the other line. We next say 5 times 6=30+8+9 (figures between the lines)=47. Write down 7 and carry 4. Next, 5 times 7=35+4 (to carry)+4+6 (between lines)=49. Write down 9 and carry 4. Next, 5 times 4=20+4 (to carry)+3 (between lines) =27. Write down 7 and carry 2 to the product of 5 times 3=17, which when written below completes the product. Proceed in like manner with any number of figures.

EXAMPLES FOR PRACTICE.

	174	1547
86	285	836
47	87	928
60	139	464
4042	49590	1293292

This method saves writing, on an average, more than half the figures of the partial products and requires no extra mental effort. As it is general in its application to all numbers we see no reason why it should not be acquired by every ambitious student.

IMPROVED CROSS MULTIPLICATION.

The method of cross multiplication, while requiring considerable mental effort where large numbers are used, is quite simple and easy when the multiplier contains only **two** figures. As the calculations of actual business usually consist of small numbers it will repay any one to learn to multiply **two** figures by **any number** of figures in a **single line.** After this has been mastered we arrange a method of relieving the mind of adding the partial products mentally when larger multipliers are used.

Short Method.		Ordinary Method.
Multiply	54	54
By	42	42
	2268	108
		216
		2268

EXPLANATION.—First, 2 (units)×4 (units)=8. Write it in the answer. Next, 2 (units)×5 (tens)=10, to which add the product of 4 (tens) by 4 (units), which gives 26. Write down 6 and carry 2 to the product of 4 (tens) by 5 (tens), which gives 22 and completes the product.

EXAMPLES FOR PRACTICE.

63	86	94	68
52	41	23	53
3276	3526	2162	3604

Short Method.		Ordinary Method.
Multiply	7365	7365
By	42	42
	309330	14730
		29460
		309330

EXPLANATION.—The first and second figures of the answer are found as in the preceding examples. We next have 2×3+4×6=30, to which we add the carrying figure (3), making 33. Write down 3 and carry 3. Next, 2×7+4×3=26. 26+3 (carrying figure)=29. Write down 9 and carry 2. The units figure (2) of the multiplier having been multiplied by the 7, or last figure of the multiplicand, it is not used further. Next multiply 4 by 7, to which we add the carrying figure (2), which gives 30 and completes the product.

EXAMPLES FOR PRACTICE.

325	7324	12345679
64	93	18
20800	681132	222222222

If the student will practice adding **pairs** of figures, as in the grouping method of addition, this, in connection with these rules, is the whole secret of rapid multiplication. We will now show how to multiply **any number** by **three** figures, making only **one** line of figures besides the answer.

Short Method.		**Ordinary Method.**
Multiply	964	964
By	832	832
	308	1928
	802048	2892
		7712
		802048

EXPLANATION.—We first multiply 964 by 32, as in the preceding examples, but instead of writing the entire product below the second line we only place the tens and units figures there (48), and the remaining figures (308) of the product we write one place to the left between the lines. We next multiply 964 by the hundreds figure (8) and add the figures between the lines in their regular order, as follows: 8×4=32, to which we add the figure (8) between the lines, which gives 40. Write down 0 and carry 4. Next 8×6=48+4 (carrying figure) +0 (between lines)=52. Write down 2 and carry 5. 8×9=72+5 (carrying figure) +3 (between lines)=80, which completes the product.

EXAMPLES FOR PRACTICE.

817	9134	83163
623	752	942
187	4749	34928
508991	6868768	78339546

The product of any number of figures by **four** figures can be obtained by multiplying in pairs or by making two lines of partial products, which is much easier, as follows:

Short Method.		Ordinary Method.
Multiply	**67183**	**67183**
By	**9452**	**9452**
	34935	**134366**
	26873	**335915**
	635013716	**268732**
		604647
		635013716

EXPLANATION.—We leave space between the lines for two lines of figures. 67183×52 (same as in preceding examples)=3493516. Write 16 in the answer and 34935 one place to the left between the lines. Next, 67183×4=268732. Add the units figure (2) to the first figure between the lines (5) we get 7, which should be written in the answer and the remaining figures, 26873, written between the lines one place to the left. We next multiply 67183 by 9, to which we add as we proceed the figures between the lines, commencing, as previously explained, with those over the first vacant space in the answer, as follows: 9×3=27. 27+3+3 (between lines)=33. Write down 3 and carry 3 to the product of 9×8, which gives 75. 75+7+9 (between lines)=91. Write down 1 and carry 9 to the product of 9×1, which gives 18. 18+8+4 (between lines)=30. Write down the 0 and carry 3 to the product of 9×7, which gives 66. 66+6+3 (between lines)=75. Write down 6 and carry 7 to the product of 9×6 which gives 61. 61+2 (between lines) =63, which when written below completes the product.

EXAMPLES FOR PRACTICE.

8534	19438
5326	8951
2218	9913
2561	17494
45452084	173989538

The beauty and value of this system of rapid multiplication is apparent to any person who desires to occupy a position in business above the ordinary. It is general in its application to all numbers, and any child who can master the usual method of long division can certainly acquire it.

NOTE.—For special rules in multiplication see page 30.

ADDING METHOD of DIVISION.

The ordinary method of long division requires us to write down the product of the divisor by each answer figure, which is quite lengthy and tedious. The method of subtracting each product *mentally* has never been popular in this country, although in France and other countries it is the almost universal method. Subtracting the products mentally in this manner is quite difficult, as the processes of subtraction and multiplication are so closely interwoven that confusion is liable to result in carrying. By the **adding method** the subtractions are made by addition, as explained under the head of subtraction. It will be found quite an easy process, and as it requires only about one-half the figures of the ordinary method it should be acquired and used by every accountant.

As this method differs from the ordinary method only in the process of subtraction the entire principle is embodied in the following example:

Divide 789 by 91.

Adding Method.

```
91)789(8
     61 Rem.
```

Ordinary Method.

```
91)789(8
   728
   ---
    61 Rem.
```

EXPLANATION.—We observe that 91 is contained in 789 *eight* times, so we write 8 in the answer, multiply the divisor by it, and place below the figures of the dividend being used the figures which when **added** to the product will equal the dividend. First, 8×1 (divisor figure) =8. **1** *added* to 8=9 (first dividend figure). Write down the 1. Next, 8×9 (divisor figure)=72; **6** *added* to 72=78. Write down the 6, which completes the remainder.

While the result is the same there is quite a difference in the mental work of finding a figure which will make a number equal another number and subtracting one number from another. When this principle is applied to long division we think of multiplying and *adding* instead of multiplying and *subtracting*, which are two separate processes.

Divide 8742 by 61.

Adding Method.

```
61)8742(143
   264
    202
     19 Rem.
```

Ordinary Method.

```
61)8742(143
   61
   ---
   264
   244
   ---
    202
    183
    ---
     19 Rem.
```

EXPLANATION.—We first observe that 61 is contained once in 87. Write 1 in the answer. 1×1=1; **6** *added* to 1=7 (dividend figure). Write down the 6. 1×6 (divisor figure)=6. **2** *added* to 6 (dividend figure)

=8). Write down 2, which leaves a partial remainder of 26. Bring down 4 (dividend figure), and we have 264. 61 is contained *four* times in 264, write 4 in the answer and proceed as before. 4×1 (divisor figure) =4; **0** *added* to 4=4 (dividend figure), so we write down 0. 4×6 (divisor figure) =24. **2** *added* to 24=26, so we write down 2. We now have a partial remainder of 20, to which we annex the next figure of the dividend (2), and we have 202. 61 is contained in 202 *three* times. Write 3 in the answer. 3×1 (divisor figure)=3. It will now be observed that the first right-hand figure of the partial remainder is 2, which is smaller than our three obtained in this manner, so, according to the method of subtraction by *adding* (see page 18), we must add the least figure to 3 which will make the unit figure of their sum equal 2. **9** is the figure which when added to 3=12. We write down 9 and carry 1. 3×6 (divisor figure)=18, to which we add the 1 (carrying figure), which gives 19. **1** *added* to 19=20 (figures above in partial remainder). Write down 1 beneath and we have the complete remainder (19).

As the method of producing each partial remainder is the same, we will avoid tiresome repetition by taking one more example with only one figure in the quotient, which should make the method plain to any one.

Divide 78287 by 8465.

Adding Method.	Ordinary Method.
8465)78287(9	**8465)78287(9**
2102 *Rem.*	**76185**
	2102 *Rem.*

Explanation.—We assume 8465 to be contained in 78287 *nine* times. 9×5=45. **2** *added* to 45=47. Write down 2 and carry 4. 9×6 (divisor figure)=54+4 (carrying figure)=58. **0** *added* to 58=58. Write down 0 and carry 5. 9×4 (divisor figure)=36+5 (carrying figure)=41. **1** *added* to 41=42. Write down 1 and carry 4. 9×8 (divisor figure)=72+4 (carrying figure)=76. **2** *added* to 76=78 (above), so write down 2, making a remainder of 2102. Perform the following example, getting the partial remainders in the same manner

Adding Method.

712)87574(122

1637

2134

710 *Rem.*

Ordinary Method.

712)87574(122

712

1637

1424

2134

1424

710

EXAMPLES FOR PRACTICE.

53)87642(1653

346

284

192

33 *Rem.*

956)76265(79

9345

741 *Rem.*

Special Short Rules in Multiplication and Division.

In addition to the rules previously shown, which are general and universal in their application, there is quite a number of special short rules in multiplication and division which depend upon the relations of certain numbers to the base of our system of notation and its multiples. Prominent among these are the aliquot parts of 10, 100, 1000, etc., which, owing to custom and convenience, enter largely into transactions of actual business.

The following table shows most of the aliquot parts used in business, and should be mastered as thoroughly as the

multiplication table. Simply memorize the aliquot parts of 100, and the aliquot parts of 10 and 1000 are either *one-tenth* or *ten times* the corresponding aliquot part of 100:

TABLE OF ALIQUOT PARTS.

50	=	*one-half*	of 100.
$33\frac{1}{3}$	=	*one-third*	of 100.
$66\frac{2}{3}$	=	*two-thirds*	of 100.
25	=	*one-fourth*	of 100.
75	=	*three-fourths*	of 100.
$16\frac{2}{3}$	=	*one-sixth*	of 100.
$83\frac{1}{3}$	=	*five-sixths*	of 100.
$12\frac{1}{2}$	=	*one-eighth*	of 100.
$37\frac{1}{2}$	=	*three-eighths*	of 100.
$62\frac{1}{2}$	=	*five-eighths*	of 100.
$87\frac{1}{2}$	=	*seven-eighths*	of 100.
$8\frac{1}{3}$	=	*one-twelfth*	of 100.
$41\frac{2}{3}$	=	*five-twelfths*	of 100.
$58\frac{1}{3}$	=	*seven-twelfths*	of 100.
$91\frac{2}{3}$	=	*eleven-twelfths*	of 100.
$6\frac{1}{4}$	=	*one-sixteenth*	of 100.
$18\frac{3}{4}$	=	*three-sixteenths*	of 100.
$31\frac{1}{4}$	=	*five-sixteenths*	of 100.
$43\frac{3}{4}$	=	*seven-sixteenths*	of 100.
$56\frac{1}{4}$	=	*nine-sixteenths*	of 100.
$68\frac{3}{4}$	=	*eleven-sixteenths*	of 100.
$81\frac{1}{4}$	=	*thirteen-sixteenths*	of 100.
$93\frac{3}{4}$	=	*fifteen-sixteenths*	of 100.

NOTE.—*Decimal Point*—Before proceeding further the student should thoroughly understand the reason for removing the decimal point, as it is of the utmost importance to a thorough knowledge of aliquot parts as applied to business. The point is always at the *left* of a decimal and on the *right* of a whole number. If the whole number has no decimal attached--being frequently omitted for convenience—it must be borne in mind whether the point is written or not that it is presumed to be on the *right* of a whole number, and that only *one point* appears in one number either whole, decimal, or mixed. The point must never be omitted in a decimal. Removing the decimal point to the *right* **multiplies** a number in a *ten-fold* ratio, and removing it to the *left* **divides** a number in a *ten-fold* ratio. Therefore if we wish to *divide* by 10 we simply remove the point *one* place to the left, by 100 *two* places,

by 1000 *three* places. For instance, we wish to *divide* 647 by 10. Here it is observed the point is not expressed, but it is presumed to be on the *right* of 647, so we remove it *one* place to the left, which gives 64.7. If we divide the same number (647) by 100 we get 6.44, and by 1000 we get .647. Now again, suppose we wish to divide the same number by 10000, we remove the point *four* places to the left. As there are only *three* figures in the number (647) we must prefix a cipher to complete the number of places and place the point to the left, which gives .0647. Proceed in like manner with any number of places. The rule when dividing or multiplying by 10, 100, or 1000 is to always remove the decimal point as many places as there are *ciphers* in the divisor or multiplier, as the case may be. Now again, suppose we wish to *multiply* 647 by 10. As 647 is a whole number we simply *annex* a cipher, which gives 6470; $647\times100=64700$, etc. Next, suppose we wish to multiply 647.53 by 10. We simply remove the point *one* place to the right, which gives 6475.3, and so on to any number of places. After thoroughly understanding how to multiply and divide by 10, 100, 1000, etc., in this manner, it becomes an easy task to multiply or divide by any aliquot part.

Divide 888 by 25.

Operation.

$$\begin{array}{r} 8.88 \\ 4 \\ \hline 35.52 \end{array} \textit{ Answer.}$$

EXPLANATION.—We first divide 888 by 100, which gives 8.88. As there are *four* 25s in 100 there are *four times* as many 25s in 888 as there aro hundreds, which gives 35.52, answer.

Divide 888 by $33\frac{1}{3}$.

Operation.

$$\begin{array}{r} 8.88 \\ 3 \\ \hline 26.64 \end{array} \textit{ Answer.}$$

Divide 321.56 by $16\frac{2}{3}$.

Operation.

$$\begin{array}{r} 3.2156 \\ 6 \\ \hline 19.2936 \end{array} \textit{ Answer.}$$

Proceed in like manner with any aliquot part. Besides those mentioned in the table there are several special cases where numbers are not aliquot parts but being so near to them that they are worked on that basis, which renders the answer sufficiently exact for all practical purposes. A perch of stone contains $24\frac{3}{4}$ cubic feet, and in some localities $16\frac{1}{2}$ cubic feet. Either of these numbers are near convenient aliquot parts; $24\frac{3}{4}$ being nearly 25, or *one-fourth* of 100, and $16\frac{1}{2}$ nearly $16\frac{2}{3}$, or *one-sixth* of 100.

Suppose we wish to find the number of large and small perches of stone in a wall which contains 1150 cubic feet.

Operation.	**Operation.**
11.50	11.50
4	6
46.00	69.00
.46	.69
46.46+*Large perches.*	69.69+*Small perches.*

EXPLANATION.—We first divide 1150 by 100 and multiply by 4, which gives (46) the number of times 25 is contained in the number. Our divisor is $24\frac{3}{4}$. By adding $\frac{1}{100}$ of the quotient (46) to itself we get 46.46 perches, which is true to the second decimal figure. With small perches $16\frac{1}{2}$ is so near $16\frac{2}{3}$ that we divide by $16\frac{2}{3}$, as shown above, and add $\frac{1}{100}$ of the quotient, as with large perches. If greater accuracy is required the answer can be easily found to any required place by continuously removing the quotient *two* places to the right and adding the results.

The number 144 occurs as a divisor quite frequently, especially in square and cubic measure and in buying and selling by the gross. Thousands of different kinds of articles of hardware, notions, etc., are sold at wholesale by the gross. As 144 is just a little more than *one-seventh* of 1000, multiplying the cost per gross in cents by .007 affords a valuable short rule for finding the price of a *single article* in cents. A little practice will enable the student to get the answer mentally in an instant.

Purchased a lot of tooth brushes at $19.00 per gross. Find the cost of a single brush.

Short Method.	**Ordinary Method.**
1.900 = { *Gross price in cents*÷1000.	144)19.00(.13+ *Cents apiece.*
7	144
13+ *Cents apiece.*	460
	432
	28

Explanation.—As 144 is a little *more* than one-seventh of 1000 the answer will always be slightly too large, but on articles that cost as high as $144.00 per gross the error will not amount to a cent. In business the object is always to get the answer to the *nearest* cent, and therefore a rule of this kind, while not mathematically correct, will answer every requirement.

When dividing by such numbers as 60, 700, 2000, etc., never commit the folly of carrying the ciphers through the calculations by long division. A few examples will make this clear.

In 43731 pounds of wheat how many bushels?

Operation.		**Operation.**
60)4373.1	**or**	60)4373.1
$728\frac{51}{60}$ *Bus.*		728.85 *Bus.*

As there are 60 pounds in a bushel of wheat our divisor is 60. Removing the point one place to the left in the dividend we get 4373.1 the number of times 10 is contained. As there are *six tens* in 60, *one-sixth* of the number of *tens* is the answer. Simply divide by 6 until the decimal point is reached, where we have a remainder of 5, to which we annex the figure (1) on the right of the decimal, which gives 51, the remainder or number of odd pounds. It can be carried through by decimals in the same manner, as shown on the right.

In 53345 pounds of hay how many tons of 2000 pounds each?

Operation.

2000)53.345

$26\frac{1345}{2000}$ *Tons.*

or

Operation.

2000)53.345

26.6725 *Tons.*

Removing the point *three* places to the left divides by 1000. Simply divide this quotient by 2 and we have the number of times 2000 is contained. Always remove the point to the left as many places as there are ciphers on the right of the divisor.

Divide 98746 by 4300.

Operation.

```
4300)987.46(22 4146/4300
      127
       41
```

or

Operation.

```
4300)987.46(22.96+
      127
       414
        276
         18
```

Explanation.—As there are two ciphers on the right of the divisor we remove the point two places in the dividend, and as the divisor contains more than one significant figure (43) we must employ long division (see adding method of division on page 27). We divide until we come to the point in the dividend and find we have a partial remainder of 41, to which we annex the figures on the right of the point in the dividend which gives 4146, the full remainder. The answer is obtained in decimal form in example on the right.

MULTIPLYING BY ALIQUOT PARTS.

(See table page 31.)

Multiply 743 by 25.

Operation.

4)74300

18575 *Answer.*

Explanation.—By annexing two ciphers to 743 we multiply it by 100. As we wish to multiply 743 by 25, or *one-fourth* of 100, we simply take *one-fourth* of 74300=18575, answer.

EXAMPLES FOR PRACTICE.

Multiply 9576 by $33\frac{1}{3}$.

Operation.

3)957600

319200 *Answer.*

Multiply 4325 by $16\frac{2}{3}$.

Operation.

6)432500

$72083\frac{1}{3}$ *Answer.*

In addition to the method of aliquot parts there are several short special rules in multiplying which deserve attention.

To multiply by any number of similar figures, as 11, 111, 4444, 88888, etc.:

$$\begin{array}{lr} \textit{Multiply} & 5324 \\ \textit{By} & 11 \\ \hline & 58564 \end{array} \textit{ Answer.}$$

EXPLANATION.—Write the units figure (4) of the multiplicand in the answer. Add the *units* (4) and *tens* (2) and write their sum (6) in the answer. (If the sum is more than 9 we place down the *unit* figure of the sum and carry the *tens.*) Next, the sum of *tens* (2) and *hundreds* (3) is 5. Write it down. Next, the sum of *hundreds* (3) and *thousands* (5) is 8. Write it down, and bringing down the thousands we have 58564, answer.

$$\begin{array}{lr} \textit{Multiply} & 5324 \\ \textit{By} & 111 \\ \hline & 590964 \end{array} \textit{ Answer.}$$

EXPLANATION.—First bring down the 4. Next, 4+2=6. Write it down. It must be observed that as we have three figures in the multiplier we must now add together *three* figures of the multiplicand until we get to the extreme left, as follows: 4+2+3=9. Write it down. Now drop the first figure (4), and add the next three figures. 2+3+5=10. Write down 0 and carry 1. Next we say 3+5=8+1 (carrying figure)=9. Write it down. Next, bring down the last figure (5) and the product is complete.

$$\begin{array}{lr} \textit{Multiply} & 5324 \\ \textit{By} & 1111 \\ \hline & 5914964 \end{array} \textit{ Answer.}$$

EXPLANATION.—First bring down 4. Next, 4+2=6. Next, 4+2+3=9. We now have *four* figures in the multiplier, and consequently must now add *four* figures of the multiplicand, as follows: 4+2+3+5=14. Write down 4 and carry 1. Next, drop first figure (4) and we have 2+3+5=10, together with carrying figure, we have 11. Write down 1 and carry 1. Next, 3+5=8+1 (to carry)=9. Next, bring down last figure and the product is complete.

$$\begin{array}{lr} \textit{Multiply} & 5324 \\ \textit{By} & 7777 \\ \hline & 5914964 \\ & 7 \\ \hline & 41404748 \end{array} \textit{ Answer.}$$

EXPLANATION.—We first multiply as in the preceding example, which gives the product of 5324 by 1111. As 7777 is *seven* times 1111, we simply multiply by one 7. If the multiplier was composed of 8s we would multiply by one 8, etc.

The product of any number by any number of *nines*, like 9, 99, 999, etc., can be obtained in two ways by *subtraction*, as follows:

Multiply 842675 by 999999.

First Short Method.		Second Short Method.
842675000000	or	999999
842675		842675
842674157325		842674157325

EXPLANATION.—By the first process we annex as many *ciphers* to the multiplicand as there are *nines* in the multiplier and subtract the multiplicand from the number thus obtained.

The second process is preferable when the factor which contains the *nines* is equal to or larger than the other factor. In such cases subtract the other factor, *less one*, from the number of *nines*, and to the remainder thus obtained prefix the same factor, *less one*.

To multiply *two figures* by *two figures* whose *units* sum is *ten* and *tens* are alike.

Multiply 96 by 94 and 65 by 65.

Operation.	Operation.
96	65
94	65
9024	4225

EXPLANATION.—Take the first example. 4 (units)×6 (units)=24. Write down 24. Add 1 to either *tens* figure, which gives 10, and multiply it by the other *tens* figure, which gives 90, and when prefixed to 24=9024, answer. When the product of the *units* by the *units* does not exceed 9, fill the *tens* place of the answer with a cipher, as for instance, 89×81=7209, etc.

This principle can be extended to all numbers of two figures whose units add to 10 and whose *tens* figures are *unlike*, as follows;

96	85	78	93
84	75	52	47
8064	6375	4056	4371

EXPLANATION.—Take the first example we say 4 (units)×6 (units)=24. Write down 4 and carry 2. Next substract the smaller *tens* (8) from the larger *tens* (9) which gives 1. Multiply the remainder (1) by the *units* figure (4) of the smaller factor (84) which gives 4, to which we add the carrying figure (2), which gives 6. Write it down. Next add 1 to the *upper ten* and multiply by the lower, which gives 80 and completes the product. This also applies to mixed numbers the sum of whose fractions equal a unit.

There are hundreds of other special short rules in multiplying and squaring numbers, but we will not include them here. Rules similar to the one preceding are of but little practical value, except when the student is on the lookout for every little mental help, and in teaching, if presented properly instead of confusing, they will not fail to interest the pupil.

Before learning the special rules we advise careful and diligent study of the *general* short methods which are the prime features of this work.

Contracted Method of Multiplication of Decimals.*

It seldom occurs that strict accuracy in the answer of a business example is required further than *cents* or hundredths place; therefore the old method which compels us to produce *unnecssary* figures to obtain the *necessary* ones, is a waste of time and mental labor. Examples containing

* NOTE.—The method we present here is far superior to the old form of this rule which required us to *reverse* one of the factors, which made it very objectionable for teaching or for business.

whole numbers only are few in proportion to those involving fractions, and the fractions of business are not of large denomination or of great variety. Common fractions of bushels, pounds, yards, feet, etc., of various commodities, and their respective prices can be easily changed into decimal form or memorized (see table of aliquot parts, page 31), after which by this process of multiplication only such portions of either factor necessary to produce the answer sufficiently exact need be used. The operations of interest, discounts, averaging accounts, measurements, etc., are wonderfully facilitated.

Find the product of 432.56 by .7 to the nearest hundredth.

Operation.

```
432.56
    .7
------
302.79+Answer.
```

Explanation.—By this method we always place the point of the multiplier directly beneath the point of the multiplicand. Be sure to remember that *tenths* place in both factors is our guide or dividing line. The *tenths* figure of the *multiplicand* and all the figures to its left are multiplied by the *tenths* figure of the *multiplier* as in ordinary multiplication. Thus in this example we have 4325×7 to the product of which however, we must add the carrying figure (4) which is procured mentally by multiplying 7 by 6 (the hundredths or rejected figure of the multiplicand). It is immaterial where the answer is written beneath as we always have two figures to point off. It is best, however, to preserve the form indicated in these examples as the points of the partial products will fall under those of the multiplier and multiplicand

Find the product of 432.56 by .07 to the nearest hundredth.

Operation.

```
432.56
   .07
------
 30.28—Answer.
```

Explanation.—We now observe that the *tenths* place of the multiplier is 0, but it serves as our guide in every instance. We now simply

multiply 432 by 7, but first we get our carrying figure from the first rejected figure (5). We say $7 \times 5 = 35$. Before proceeding further let us dwell upon the importance of getting these carrying figures equalized, because the correctness of our product in a long example may be uncertain unless we use proper judgment as we proceed. It was seen in the preceding example that our carrying figure was 4 or the tens figure of the product of 7×6. We took the 4 because 42 is nearer 40 than 50. Now in this case 35 is midway between 30 and 40, and we must always give the upper number preference, especially if there are figures on the right of the rejected figure. Therefore we will now call the carrying figure 4 instead of 3. So we simply say 432 multiplied by 7 to which we add the carrying figure. The answer 30.28 is slightly too large, but it is to the nearest hundredth.

Find the product of 432.56 by .007 to the nearest hundredth.

Operation.

$$\begin{array}{r} 432.56 \\ .007 \\ \hline 3.03\text{—}\textit{Answer.} \end{array}$$

EXPLANATION.—The first 0 rejects the 5 above, and the second 0 rejects the 2 above, consequently we have 43×7 to which we add 2 (to carry). The ingenious student can look back over the rejected figures 256 and perceive that the carrying figure is really nearer 2 than 1. A little good mental drill on these carrying figures is absolutely necessary.

Find the product of 432.56 by .0007 to the nearest hundredth.

Operation.

$$\begin{array}{r} 432.56 \\ .0007 \\ \hline .30+\textit{Answer.} \end{array}$$

EXPLANATION.—First 0 rejects 5, second 0 rejects 2, third 0 rejects 3, consequently we say $7 \times 4 = 28$, to which we add carrying figure (2), which gives .30, answer.

Find the product of 432.56 by .00007 to the nearest hundredth.

Operation.

$$\begin{array}{r} 432.56 \\ .00007 \\ \hline .03\text{—}\textit{Answer.} \end{array}$$

EXPLANATION.—First 0 rejects 5, second 0 rejects 2, third 0 rejects 3, fourth 0 rejects 4, but as our carrying figure is always *hundredths* we must not think our answer is 0. $7 \times 4 = 28$, which is nearer 30 than 20, so we carry 3, making 3 hundredths, answer.

If the student will closely follow these instructions, with some practice it will be found an easy road.

We will now embody all these separate explanations in one example which will show the full working of the rule.

Multiply 432.56 by .77777 to the nearest hundredth.

Short Method.	**Ordinary Method.**
432.56	432.56
.77777	.77777
30279	302792
3028	302792
303	302792
30	302792
3	302792
336.43	336.4321912

Multiply 432.56 by 7.

Operation.

```
 432.56
      7.
 -------
3027.92
```

EXPLANATION.—This example is given to show that when we have whole numbers in the multiplier, the *tens* figure goes by the *hundredths* of the multiplicand and all on its left, etc. It must be remembered that this rule does not save figures when only whole numbers occur, but it avoids useless figuring when decimals are involved.

We will now present a few appropriate examples to illustrate the practical value of the rule.

Find the cost of $17\frac{5}{8}$ yards of cloth @ $13\frac{1}{2}$¢ per yard, and $91\frac{3}{4}$ pounds of butter @ $7\frac{3}{4}$¢ per pound.

Operation.

```
17.625
  .135
------
 176
  53
   9
-----
$ 2.38—Answer.
```

Operation.

```
91.75
 .0775
------
 642
  64
   5
-----
$7.11+Answer.
```

The interest of $1 @ 6% for 47 days is .00783+ * Find the interest on $93.25 and $941.75 at the same rate and time.

Operation.

```
$93.25
  .00783
--------
   65
    7
------
$.72+Answer.
```

Operation.

```
$941.75
  .00783
--------
  659
   75
    3
------
$7.37+Answer.
```

The circumference of any circle is equal to the diameter multiplied by 3.14159+; find the circumference of a circle whose diameter is 5.85 inches.

Operation.

```
 3.14159
 5.85
--------
1571
 251
  16
------
18.38—Inches, diameter.
```

* For short methods of getting the interest of $1 at any rate per cent for any length of time, see "INTEREST."

FRACTIONS.

ADDITION OF FRACTIONS.

Find the sum of $\frac{3}{7}$ and $\frac{5}{11}$.

Operation.

$$\overset{33}{\frac{3}{7}} \quad + \quad \overset{35}{\frac{5}{11}} = \frac{68}{77} \text{ \textit{Answer.}}$$

EXLANATION.—We multiply the denominator (7) of the first fraction by the numerator (5) of the second fraction, which gives 35. Next we multiply the numerator (3) of the first fraction by the denominator (11) of the second fraction, which gives 33. We now unite these products (35+33=68), which gives the numerator of the answer. The denominator of the answer is the product of the denominators (7×11=77).

Find the sum of $\frac{4}{7}$, $\frac{2}{3}$ and $\frac{3}{4}$.

Operation.

$$\overset{48}{\frac{4}{7}} + \overset{56}{\frac{2}{3}} + \overset{63}{\frac{3}{4}} = \frac{167}{84} \text{ \textit{Answer.}}$$

EXPLANATION.—We take the numerator (4) of the first fraction and multiply it by the product of the denominators of the other fractions (3×4=12). 12×4=48. Write it down. We next multiply the second numerator (2) by the product of the right and left-hand denominators (7×4=28) 28×2=56. Write it down. Next, the third numerator we multiply by the product of the first two denominators (3×7=21), 21×3=63, which completes the partial sums. We add them and get 167, which written over the product of the denominators (7×3×4=84) completes the sum.

SUBTRACTION OF FRACTIONS.

From $\frac{7}{8}$ take $\frac{2}{3}$.

Operation.

$$\overset{21}{\frac{7}{8}} \quad - \quad \overset{16}{\frac{2}{3}} = \frac{5}{24} \text{ \textit{Answer.}}$$

EXPLANATION.—First we multiply the denominator (8) of the subtrahend by the numerator (2) of the minuend, which gives 16. Next, we multiply the numerator (7) of the subtrahend by the denominator (3) of the minuend, which gives 21. We then take the smaller product (16) from the greater (21), which leaves 5, the numerator of the answer. The product of the denominators (24) is the denominator of the answer.

From $17\frac{3}{8}$ take $12\frac{9}{11}$.

Operation.

$$\begin{array}{l} 17\frac{3}{8} + \frac{2}{11} \\ 12\frac{9}{11} \\ \hline \;\;4\frac{49}{88} \text{ \textit{Answer.}} \end{array}$$

EXPLANATION.—We observe that the fraction of the minuend is larger than the fraction of the subtrahend. In such cases add *one* to the minuend and deduct it from the subtrahend. $12+1=13$. $17-13=4$, the whole number of the answer. Deduct the fraction of the minuend from *one* and add the difference to the upper fraction, thus: $\frac{11}{11}-\frac{9}{11}=\frac{2}{11}$ $\frac{3}{8}+\frac{2}{11}=\frac{49}{88}$, the fraction of the answer.

MULTIPLICATION OF MIXED NUMBERS

When the answer is desired in correct fractional form we may employ the following general method without reducing each factor to an improper fraction:

Multiply $94\frac{2}{3}$ by $63\frac{7}{8}$.

Operation.

$$\begin{array}{r|l} 6\,5\,8 & 94\frac{2}{3} \\ 1\,2\,6 & 63\frac{7}{8} \\ \hline & 5922\;\;14 \\ & 82\;\;6 \\ & 42\;\;0 \\ \hline & 6046\frac{20}{24} \text{ \textit{Answer.}} \end{array}$$

EXPLANATION.—First find the product of the whole numbers, which can always be done in a single line (see page 24). We write the product

(5922) beneath, and to the right of this line write the product of the numerators of the fractions ($2\times7=14$). Next multiply the numerator (7) of the lower fraction by the upper whole number (94), which gives 658, which write on the left of the upper number. Now divide the product thus obtained by the denominator (8) of the lower fraction, which gives 82 and a remainder of 2. Write the 82 in the whole number column and the remainder (2) we multiply by the upper denominator (3), giving a product of 6, which we write under 14 in the fraction column. Next, multiply the lower whole number (63) by the numerator (2) of the upper fraction, which gives 126. Write it on the left. Now divide 126 by the denominator (3) of the upper fraction, which gives 42 with no remainder. Write 0 in the fraction column. Now simply add the partial products and our product is complete. If the partial products of the fractions amount to more than 1 carry the excess to the whole numbers.

DIVISION OF MIXED NUMBERS.

Divide $49\frac{3}{11}$ by 9.

Operation.

$$\begin{array}{r} 9\,)\,49\frac{3}{11} \\ \hline 5\frac{47}{99} \end{array}\ \textit{Answer.}$$

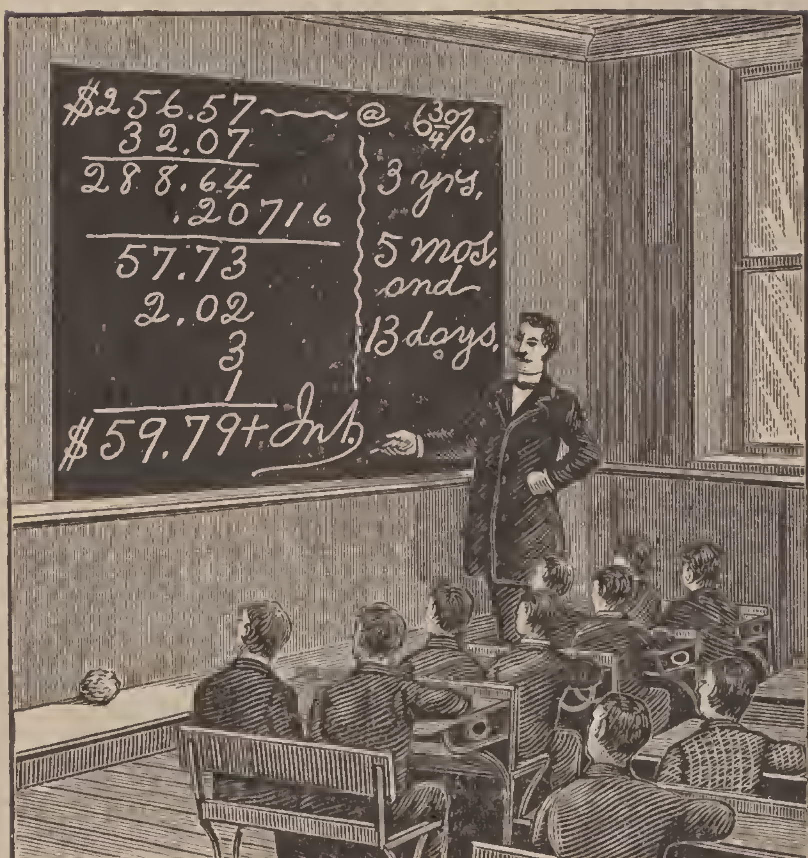
EXPLANATION.—In the usual method both dividend and divisor are reduced to the same denomination. In cases where the divisor is a whole number this is unnecessary. We proceed as follows: 9 is contained 5 times in 49 with a remainder of 4. Write down 5, and to produce the fraction of our quotient we multiply the remainder (4) by the denominator (11), which gives 44; to this we add the numerator (3) and we have 47, the numerator of the quotient. The product of the divisor by the denominator is the denominator (99) of the answer.

EXAMPLES FOR PRACTICE.

Divide $732\frac{8}{11}$ by 9, and $6857\frac{7}{8}$ by 12.

$$\begin{array}{r} 9\,)\,732\frac{8}{11} \\ \hline 81\frac{41}{99} \end{array} \qquad \begin{array}{r} 12\,)\,6857\frac{7}{8} \\ \hline 571\frac{47}{96} \end{array}$$

SIMPLE INTEREST.



GENERAL SHORT METHOD.

There are a great many *special* short rules for computing interest all hinging upon the relations the time, per cent, and principal bear to the year or month assumed as a base.

It is commonly assumed in computing interest that a month contains 30 days and a year 360 days. When the principal, time, and rate per cent are *prime* to the number of days or months in a year all special short rules are at a

disadvantage, and really most of them, as shown in calculators and arithmetics are in many cases longer and more tedious than the old method of aliquot parts.

We will discuss the merits of the best of these further on, but will now proceed to explain a method which is practical and general in its adaptation to all possible cases and saves a vast amount of useless figuring.

If we could instantly find the interest of $1 at any rate per cent for any length of time mentally, further than that the calculation would be easy. Per cents such as $6\frac{1}{2}$, 7, 11, etc., are *prime* to the number of days in a year, and the mental operation in producing the interest of $1 is too great; therefore we assume a *base rate* which is *not* prime and make corrections from it, more or less, as the case may be. The corrections can be made from the interest of $1 or from the principal.

In assuming 360 days a year the rates of 3 and its multiples, as 6, 9, 12, etc., offer a number of easy routes in treating the time and producing the interest of $1 mentally.

The most convenient *base rate* to use is 6, as it is the legal rate in most States, and besides, the interest of $1 for any length of time can be produced mentally in an instant, In the first place learn the decimal equivalents of all the fractions of *sixths*, as follows:

$$\frac{1}{6} = .16+$$
$$\frac{2}{6} = .33+$$
$$\frac{3}{6} = .5$$
$$\frac{4}{6} = .66+$$
$$\frac{5}{6} = .83+$$

Find the interest of $1 for 47 days.

EXPLANATION.—We divide the time in days by 6, and when the time is less than 60 days we place down a point and two ciphers (.00). $47 \div 6 = 7$ and 5 remainder. We write the 7 to the right of the .00 and we have .007, to which we annex the decimal equivalent of $\frac{5}{6}$ (83+) which gives .00783+ answer. The decimal extended would be .007833333+.

Find the interest of $1 @ 6% for

13	days.	*Answer*	$ .00216+
29	"	"	.00483+
17	"	"	.00283+
16	"	"	.00266+
18	"	"	.003
24	"	"	.004
48	"	"	.008
53	"	"	.00883+
49	"	"	.00816+
37	"	"	.00616+
59	"	"	.00983+

NOTE.—When the days are less than 6 the interest is simply one of the decimal equivalents of *sixths* with three ciphers.

Find the interest of $1 @ 6% for 73 days.

EXPLANATION.—When the days are less than 600 and more than 60 we have only one cipher before writing the significant figures of the interest. 73 days are more than 60, therefore we write down a point and a cipher (.0). 73÷6=12 and a remainder of 1. We annex 12 to the cipher, gives .012, to which we annex the decimal of $\frac{1}{6}$ (.16+), which gives .01216+ answer.

Find the interest @ 6% for

60	days.	*Answer*	$ .01
63	"	"	.0105
79	"	"	.01316+
84	"	"	.014
93	"	"	.0155
314	"	"	.05233+
423	"	"	.0705

When the time consists of years, months, and days, we have the following:

Find the interest of $1 for 2 years, 8 months, and 12 days.
Answer, $.162

EXPLANATION.—We multiply the years by 6 and to the product add one-half the number of months. Thus, 2 (yrs.)×6=12. 8 (mos.)÷2=4. We now add 12 and 4, which gives 16. This result is always *cents*; therefore we place a point, and on the right place the 16. We now take *one-sixth* of the number of days (12), which gives 2, and when annexed to .16 we have $.162, answer.

Find the interest of $1 @ 6% for 3 years, 9 months, and 19 days. *Answer*, $.22816+

EXPLANATION.—In this example the months are *odd*, and a little hint here will enable us to avoid using a fraction. We proceed as in the preceding example. We say 3 (yrs.)×6=18. Take one-half of 9 (mos.) we have 4½. We say 18+4=22 (cents), which we write down, and the fraction of ½ we call 30 days (or 3 tens), and when added to the number of days (19) we get 49. 49÷6=8 and a remainder of 1. Write down 8 to the right of the .22, and we have .228, to which we annex in decimal form the remainder of *sixths* (1), we have .22816+, answer.

Find the interest of $1 @ 6% for

5	years,	4	months,	and	17	days,	$.32283+	*Answer.*
2	"	9	"	"	11	"	.16683+	"
1	"	7	"	"	18	"	.098	"
3	"	4	"	"	26	"	.14433+	"
		8	"	"	12	"	.042	"
		9	"	"	21	"	.0485	"

When the interest of $1 at any other rate per cent is desired we first find the interest at 6% and then for each whole per cent above 6 we add $\frac{1}{6}$ more and for each whole per cent less than 6 we deduct $\frac{1}{6}$ of the interest. Thus, if we desire the interest at 7% we would add $\frac{1}{6}$, if 8%, $\frac{2}{6}$ or $\frac{1}{3}$, if 9%, $\frac{3}{6}$ or $\frac{1}{2}$, etc. If we add or deduct $\frac{1}{6}$ for each whole per cent we should add or deduct $\frac{1}{12}$ for each one-half per cent and $\frac{1}{24}$ for each one-fourth per cent, etc.

Find the interest of $1 @ 7% for 2 years, 4 months, and 18 days.

Operation.

$$\begin{array}{r} 6\,).143 \\ .02383+ \\ \hline .16683+ \end{array}$$

EXPLANATION.—We first find the interest at 6%, as in the preceding examples, and we get .143. *One-sixth* of .143=.02383+ which added to .143=.16683+. Now, suppose we wish to find the interest at 7½%. We observe that there are *three* halves above 6, consequently we should add $\frac{3}{12}$ or ¼ of the interest @ 6%, etc.

Find the interest of $1 for

42 days	@	8 %	*Answer*	$ .00933+
73 "	"	9 %	"	.01825
213 "	"	7½ %	"	.04437+
63 "	"	5 %	"	.00875
21 "	"	4 %	"	.00233+
83 "	"	4½ %	"	.01037+

We are now prepared to compute the interest on any sum of money.

Find the interest of $241.50 @ 6% for 47 days.

Operation.

```
$241.50
     .00783+ Int. of $1.
-----------
     169
      19
       1
    ----
  $1.89+ Answer.
```

EXPLANATION.—Of course any rule for multiplying the principal by the interest of $1 will produce the complete interest, but the contracted method of multiplying decimals (see pages 38 to 42) will be of great assistance in disposing of useless figures, as our answer need be correct only to the *nearest* cent. As the *time* of most notes, etc., is less than a year the interest of $1 is usually a very easy multiplier by this method, no difference how difficult it may appear by the old method of multiplying decimals.

We can if we choose always use the interest of $1 @ 6% as a multiplier and add to or deduct from the *principal* such portion of it as the per cent is greater or less than 6. Therefore, if the student will practice on the method of instantly getting the interest of $1 @ 6% mentally, this, together with

the short method of multiplying, will undoubtedly afford the shortest route to the required interest. Of course there may be cases where some special short rule which applies only to a few examples will produce the answer with fewer figures, but in the vast majority of business examples this will be found the one to rely upon.

Find the interest of \$248.75 for 1 year, 4 months, and 11 days at $7\frac{1}{2}\%$.

Operation.

$$
\begin{array}{r}
\$248.75 \\
62.19 \\
\hline
\$310.94 \\
.08183+ \\
\hline
2487 \\
31 \\
25 \\
1 \\
\hline
\$25.44+\ \textit{Answer.}
\end{array}
$$

EXPLANATION.—$7\frac{1}{2}\%$ is $1\frac{1}{2}\%$ greater than 6%. For each half above 6% we add $\frac{1}{12}$, and as there are three halves above we add $\frac{3}{12}$ or $\frac{1}{4}$ of the *principal* to itself. One-fourth of \$248.75 is \$$62.18\frac{3}{4}$, or nearly \$62.19. Always add to the nearest cent in this manner. \$248.75 + \$62.19 = \$310.94. In other words, it will require \$310.94 @ 6% to earn as much money at interest as \$248.75 could at $7\frac{1}{2}\%$. We find the interest of \$1 @ 6% by the preceding rule is .08183 +, which we multiply by the principal—using the contracted method of decimals, and we have \$25.44 +, answer.

It is very seldom that a more difficult example than this one is encountered in actual business. The idea of making the correction from the principal is, we believe, entirely original, and is to be preferred to making corrections from the interest.

EXAMPLES FOR PRACTICE.

Find the interest of \$925.75 @ 6% for 53 days, and \$78.42 @ 7% for 1 year, 7 months, and 11 days.

Operation.

$$\begin{array}{r} \$925.75 \\ .00883+ \\ \hline 740 \\ 74 \\ 3 \\ \hline \$8.17+ \end{array}$$ *Answer.*

Operation.

$$\begin{array}{r} \$78.42 \\ 13.07 \\ \hline 91.49 \\ .09683+ \\ \hline 823 \\ 55 \\ 7 \\ \hline \$8.85+ \end{array}$$ *Answer.*

ACCURATE INTEREST.

Accurate interest is based on the true year of 365 days. Interest is computed by counting the exact number of days; each day's interest being $\frac{1}{365}$ part of the interest for one year, whereas by the commercial year, so generally used in this country, the interest for each day is its $\frac{1}{360}$ part. It will thus be observed that the accurate interest for any time less than a year is slightly less than when 360 days are taken as a base. Ordinary interest can be changed to accurate interest by deducting its $\frac{1}{73}$ part. 73 being a very clumsy divisor, an easier method is to employ the following:

The ordinary interest of a certain note amounts to $182.50, what is the accurate interest?

Operation.

$$\begin{array}{r} \$182.50 \\ 2.50 \\ \hline \$180.00 \end{array} \qquad \begin{array}{r} 183 \\ 55 \\ 12 \end{array}$$

EXPLANATION.—Write down the number of dollars to the nearest unit (183). Next write down 3 times the tens, hundreds, etc., to the nearest unit, thus: 18×3=54+1 (to carry)=55, Next, 7 times the hundreds to the nearest unit, 7×1=7+5 (to carry)=12. Add these products and deduct their sum from the ordinary interest.

Special Short Interest Rules.

DECIMAL METHOD FOR DAYS.

(Assumed year 360 days.)

One-tenth part of the **principal** is the interest at	*One-hundredth* part of the **principal** is the interest at	*One-thousandth* part of the **principal** is the interest at
4 % for 900 days.	4 % for 90 days.	4 % for 9 days.
5 % for 720 days.	5 % for 72 days.	5 % for 7.2 days.
6 % for 600 days.	6 % for 60 days.	6 % for 6 days.
7 % for 514 days.*	7 % for 52 days.*	7 % for 5.2 days.*
7½ % for 480 days.	7½ % for 48 days.	7½ % for 4.8 days.
8 % for 450 days.	8 % for 45 days.	8 % for 4.5 days.
9 % for 400 days.	9 % for 40 days.	9 % for 4 days.
10 % for 360 days.	10 % for 36 days.	10 % for 3.6 days.
12 % for 300 days.	12 % for 30 days.	12 % for 3 days.

EXLANATION.—This method is founded upon the principle that if $1 in 1 year earns *6 cents* at 6%, in $\frac{1}{6}$ of 1 year—or 60 days—$1 will earn *1 cent;* therefore, $\frac{1}{100}$ part of the principal is the interest for 60 days, $\frac{1}{1000}$ of the principal the interest for 6 days, etc.

Find the interest of $321.00 @ 9% for 40 days, and on $763.25 for 60 days at 6%.

Operation.	**Operation.**
$3|21.00	$7|63.25

EXPLANATION.—Take the first example. If $1 earns 9 cents in 1 year at 9% in $\frac{1}{9}$ of 1 year, or 40 days, each dollar will earn 1 cent, or $321 will earn 321 cents. We draw a line two places to the left, which divides by 100, and we have the interest. When the number of days in a year is not exactly divisible by the per cent—such as 7, 11, etc.—the time it takes $1 to earn 1 cent, mill, etc., is fractional, and the rule in such cases is almost useless. The better plan is to use 6% as a base, and make additions or deductions as in the "General Method."

We have simply to remember that removing the point one place gives the interest for 600 days, two places for 60 days, and three places for 6 days. A line can be drawn through the principal to represent the point.

* Nearly.

Find the interest of $468 for 60 days at 7%.

Operation.

$4	68.
	78
$5	46 *Answer.*

EXPLANATION.—Drawing a line two places to the left gives the interest for 60 days @ 6%. 7% is $\frac{1}{6}$ more, so we add $\frac{1}{6}$ of the interest to itself, which gives $5.46, answer.

Find the interest of $675.36 for 19 days @ 6%.

Operation.

$	675.36 = Interest for 6 days.
	3
2	026 08 = Interest for 18 days.
	112 56 = Interest for 1 day.
$2	138 64 = Interest for 19 days.

EXPLANATION.—Drawing the line *three* places to the left, and we have the interest at 6% for 6 days. We multiply by 3 and get $2.02608, interest for 18 days. We lack a day's interest of completing the answer so we take $\frac{1}{6}$ of the interest for 6 days .67536÷6=.11256, which added to the interest for 18 days we have $2.13+, answer. Of course it is not necessary to get the answer nearer than *cents*, so we should omit useless decimals in actual business. The line can be drawn on the time as well as on the principal as follows:

Find the interest of $600 for 213 days at 6%.

Operation.

$21 | 30

EXPLANATION.—The interest of $600 for 213 days is the same as the interest of $213 for 600 days, therefore we take $\frac{1}{10}$ of the number of days, and we have 21.3 or $21.30, answer.

Find the interest of $40 for 743 days at 9%.

Operation.

7 | 43

EXPLANATION.—The interest of $1 at 9% for 1 year is 9 cents, and for $\frac{1}{9}$ of 1 year—or 40 days—is 1 cent, therefore the interest of $40 for 1 day is 1 cent, and for 743 days is $7.43.

SAVINGS BANK COMPOUND INTEREST TABLE.

Showing the amount of $1, from 1 year to 15 years, with Compound Interest added semi-annually, at different rates.

	Three Per cent.	*Four Per cent.*	*Five Per cent.*	*Six Per cent.*	*Seven Per cent.*	*Eight Per cent.*	*Nine Per cent.*	*Ten Per cent.*
½ year..........	$1.01	$1.02	$1.02	$1.03	$1.03	$1.04	$1.04	$1.05
1 year..........	1.03	1.04	1.05	1.06	1.07	1.08	1.09	1.10
1½ years.........	1.04	1.06	1.07	1.09	1.10	1.12	1.14	1.15
2 years.........	1.06	1.08	1.10	1.12	1.14	1.16	1.19	1.21
2½ years.........	1.07	1.10	1.13	1.15	1.18	1.21	1.24	1.27
3 years.........	1.09	1.12	1.15	1.19	1.22	1.26	1.30	1.34
3½ years.........	1.10	1.14	1.18	1.22	1.27	1.31	1.36	1.40
4 years.........	1.12	1.17	1.21	1.26	1.31	1.36	1.42	1.47
4½ years.........	1.14	1.19	1.24	1.30	1.36	1.42	1.48	1.55
5 years.........	1.16	1.21	1.28	1.34	1.41	1.48	1.55	1.62
5½ years.........	1.17	1.24	1.31	1.38	1.45	1.53	1.62	1.71
6 years.........	1.19	1.26	1.34	1.42	1.51	1.60	1.69	1.79
6½ years.........	1.21	1.29	1.37	1.46	1.56	1.66	1.77	1.88
7 years.........	1.23	1.31	1.41	1.51	1.61	1.73	1.85	1.97
7½ years.........	1.24	1.34	1.44	1.55	1.67	1.80	1.93	2.07
8 years.........	1.26	1.37	1.48	1.60	1.73	1.87	2.02	2.18
8½ years.........	1.28	1.39	1.52	1.65	1.79	1.94	2.11	2.29
9 years.........	1.30	1.42	1.55	1.70	1.85	2.02	2.20	2.40
9½ years.........	1.32	1.45	1.59	1.75	1.92	2.10	2.30	2.52
10 years.........	1.34	1.48	1.63	1.80	1.98	2.19	2.41	2.65
11 years.........	1.38	1.54	1.72	1.91	2.13	2.36	2.63	2.92
12 years.........	1.42	1.60	1.80	2.03	2.28	2.56	2.87	3.22
13 years.........	1.47	1.67	1.90	2.15	2.44	2.77	3.14	3.55
14 years.........	1.51	1.73	1.99	2.28	2.62	2.99	3.42	3.62
15 years.........	1.56	1.80	2.09	2.42	2.80	3.24	3.74	4.32

TIME AT WHICH MONEY DOUBLES AT INTEREST.

Rate per cent.	*Simple Interest.*	*Compound Interest.*
2........................	50 years.	35 years 1 day.
2½........................	40 years.	28 years 26 days.
3........................	33 years 4 months.	23 years 164 days.
3½........................	28 years 208 days.	20 years 54 days.
4........................	25 years.	17 years 246 days.
4½........................	22 years 81 days.	15 years 273 days.
5........................	20 years.	15 years 75 days.
6........................	16 years 8 months.	14 years 327 days.
7........................	14 years 104 days.	10 years 89 days.
8........................	12½ years.	9 years 2 days.
9........................	11 years 40 days.	8 years 16 days,
10........................	10 years.	7 years 100 days.

ONE DOLLAR LOANED 100 YEARS at Compound Interest would amount to the following sum:

1 per cent..................	$2.75	12 per cent............	$84,675.00
3 per cent..................	19.25	15 per cent............	1,174,405.00
6 per cent..................	340.00	18 per cent............	15,145,207.00
10 per cent..............	13,809.00	24 per cent........	2,551,799,404.00

Difference in Time Table.

(FIRST YEAR.)

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	32	60	91	121	152	182	213	244	274	305	335
2	33	61	92	122	153	183	214	245	275	306	336
3	34	62	93	123	154	184	215	246	276	307	337
4	35	63	94	124	155	185	216	247	277	308	338
5	36	64	95	125	156	186	217	248	278	309	339
6	37	65	96	126	157	187	218	249	279	310	340
7	38	66	97	127	158	188	219	250	280	311	341
8	39	67	98	128	159	189	220	251	281	312	342
9	40	68	99	129	160	190	221	252	282	313	343
10	41	69	100	130	161	191	222	253	283	314	344
11	42	70	101	131	162	192	223	254	284	315	345
12	43	71	102	132	163	193	224	255	285	316	346
13	44	72	103	133	164	194	225	256	286	317	347
14	45	73	104	134	165	195	226	257	287	318	348
15	46	74	105	135	166	196	227	258	288	319	349
16	47	75	106	136	167	197	228	259	289	320	350
17	48	76	107	137	168	198	229	260	290	321	351
18	49	77	108	138	169	199	230	261	291	322	352
19	50	78	109	139	170	200	231	262	292	323	353
20	51	79	110	140	171	201	232	263	293	324	354
21	52	80	111	141	172	202	233	264	294	325	355
22	53	81	112	142	173	203	234	265	295	326	356
23	54	82	113	143	174	204	235	266	296	327	357
24	55	83	114	144	175	205	236	267	297	328	358
25	56	84	115	145	176	206	237	268	298	329	359
26	57	85	116	146	177	207	238	269	299	330	360
27	58	86	117	147	178	208	239	270	300	331	361
28	59	87	118	148	179	209	240	271	301	332	362
29	—	88	119	149	180	210	241	272	302	333	363
30	—	89	120	150	181	211	242	273	303	334	364
31	—	90	—	151	—	212	243	—	304	—	365

EXPLANATION.—Find the difference of time between March 12 and October 27 in the same year. Look for March and underneath, opposite 12 in heavy-face figures, we find 71. Deduct thi from the figures corresponding in the same manner to Octobe 27 (300), and we get 229 days, answer. If a date runs into anothe year find the later date in the opposite table and proceed as before, These tables are useful in finding the

Difference in Time Table.

(SECOND YEAR.)

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.	
366	397	425	456	486	517	547	578	609	639	670	700	**1**
367	398	426	457	487	518	548	579	610	640	671	701	**2**
368	399	427	458	488	519	549	580	611	641	672	702	**3**
369	400	428	459	489	520	550	581	612	642	673	703	**4**
370	401	429	460	490	521	551	582	613	643	674	704	**5**
371	402	430	461	491	522	552	583	614	644	675	705	**6**
372	403	431	462	492	523	553	584	615	645	676	706	**7**
373	404	432	463	493	524	554	585	616	646	677	707	**8**
374	405	433	464	494	525	555	586	617	647	678	708	**9**
375	406	434	465	495	526	556	587	618	648	679	709	**10**
376	407	435	466	496	527	557	588	619	649	680	710	**11**
377	408	436	467	497	528	558	589	620	650	681	711	**12**
378	409	437	468	498	529	559	590	621	651	682	712	**13**
379	410	438	469	499	530	560	591	622	652	683	713	**14**
380	411	439	470	500	531	561	592	623	653	684	714	**15**
381	412	440	471	501	532	562	593	624	654	685	715	**16**
382	413	441	472	502	533	563	594	625	655	686	716	**17**
383	414	442	473	503	534	564	595	626	656	687	717	**18**
384	415	443	474	504	535	565	596	627	657	688	718	**19**
385	416	444	475	505	536	566	597	628	658	689	719	**20**
386	417	445	476	506	537	567	598	629	659	690	720	**21**
387	418	446	477	507	538	568	599	630	660	691	721	**22**
388	419	447	478	508	539	569	600	631	661	692	722	**23**
389	420	448	479	509	540	570	601	632	662	693	723	**24**
390	421	449	480	510	541	571	602	633	663	694	724	**25**
391	422	450	481	511	542	572	603	634	664	695	725	**26**
392	423	451	482	512	543	573	604	635	665	696	726	**27**
393	424	452	483	513	544	574	605	636	666	697	727	**28**
394	—	453	484	514	545	575	606	637	667	698	728	**29**
395	—	454	485	515	546	576	607	638	668	699	729	**30**
396	—	455	—	516	—	577	608	—	669	—	730	**31**

dates of the maturity of notes, etc. For instance, what is the date of maturity of a note to be paid 63 days from July 4? We look to July 4 and find 185 to which we add 63 and get 248. 248 is in the September column and opposite 5, which is the date required. When February 29 occurs between two dates count an additional day.

SHORT BUSINESS RULES.

MARKING GOODS BOUGHT BY THE DOZEN.

Articles of nearly every description are sold at wholesale by the *dozen* or *gross*. As purchases are usually made in a hurry it becomes a matter of prime importance to the merchant or buyer to be able to instantly find the retail selling price of a single article from the dozon or gross price.

To make	20 %,	divide the	cost per	dozen by	10			
"	$33\frac{1}{3}$	"	"	"	9			
"	50	"	"	"	8			
"	100	"	"	"	6			
"	40	"	"	"	10,	and add	$\frac{1}{6}$	itself.
"	35	"	"	"	10	"	$\frac{1}{8}$	"
"	$37\frac{1}{2}$	"	"	"	10	"	$\frac{1}{4}$	"
"	30	"	"	"	10	"	$\frac{1}{12}$	"
"	25	"	"	"	10	"	$\frac{1}{24}$	"
"	$12\frac{1}{2}$	"	"	"	10,	and substract	$\frac{1}{16}$	"
"	$16\frac{2}{3}$	"	"	"	10	"	$\frac{1}{36}$	"
"	$18\frac{3}{4}$	"	"	"	10	"	$\frac{1}{96}$	"

Bought a dozen hats at $37.50 per dozen. What is the retail price of each hat at 20% profit?

EXPLANATION.—The above table should be memorized. To make 20% profit on a single article we simply divide the cost per dozen by 10, which is done by removing the point one place to the left, and in this case we have $3.75, answer.

MARKING GOODS BOUGHT BY THE GROSS.

Small articles, such as brushes, combs, hinges, locks, notions, etc., are usually sold at wholesale by the gross.

See pages 33 and 34 for short method of dividing by 144, which will enable any one to get the cost of a *single article* mentally from the gross price. The per cent of profit can then be added.

MARKING GOODS BOUGHT BY THE NEST.

Tubs, pails, baskets, etc., are sold at wholesale by the nest. If we wish to find the proportionate cost of each, proceed as follows:

Bought 1 nest containing 5 tubs costing $6. The smallest tub contains 1 gallon, the second 2 gallons, the largest 5 gallons, etc.

Cost of 1 gallon	$ .40.
Cost of 2 gallons	.80.
Cost of 3 gallons	1.20.
Cost of 4 gallons	1.60.
Cost of 5 gallons	2.00.

EXPLANATION.—We take the sum of the gallons 1, 2, 3, 4, 5 = 15, and divide the price by it. $6.00 divided by 15 = 40 cents, the price of 1 gallon. Then simply multiply by the number of gallons in each tub. Always divide the price of the nest by the number of gallons in all the tubs and multiply the quotient by the gallons in each tub.

MERCHANTS' RULE.

Retailers frequently sell coffee, sugar, etc., at a given number of pounds per dollar, and the following will show a simple method of determining the proportionate quantity to sell for fractions of a dollar.

A merchant sells sugar @ 9¾ pounds per dollar. How many pounds will 60 cents purchase?

EXPLANATION.—Simply multiply the number of pounds sold for $1 by the cents' worth desired. Thus, 9.75×.60=5.85 pounds, answer. Hundredths and tenths of a pound can be reduced to ounces by multiplying by 16.

For short methods of **Trade Discounts**, etc., see pages 19 to 21,

Chronology or Computing Time.

The Julian Calendar, or Old Style, established by Julius Cæsar 46 B. C., was in use in all Christian countries until the year 1582, when Pope Gregory XIII decreed that in all Catholic countries the Julian Calendar should be abolished and a correction of 10 days be made, making the 5th of the month of October in that year the 15th; and also that instead of reckoning every four years a leap year the centurial years, as 1600, 1700, 1800 should not be leap years unless divisible by 400. It will thus be seen that 1600 was a leap year according to the New Style, but 1700, 1800, and 1900 are not. The next centurial leap year will be 2000. The New Style was not adopted in England and America until 1752. The Old Style is still used in Russia.

According to the Old Style every fourth year is counted a leap year and an extra day is added to February. As the true solar year does not contain exactly 365¼ days this error was the cause of the adoption of the New Style, which is sometimes called the Gregorian Calendar, but which was really devised by Aloysius Lilius, or Luigi Lilio Ghiraldi, a learned astronomer and physician of Naples, who died before its adoption. It devolved upon Clavius to make all the calculations necessary for its verification, and by whom it was completely developed and explained in a large folio treatise of 800 pages, published in 1603.

It is our mission to strip the long methods, tables, etc., of the folio treatise of their dominical letters, technical terms, etc., and to present the simplest rules for finding the day of the week of any date, Easter Sunday, and the principal church feasts depending on Easter, Age of Moon, etc.

OLD STYLE.

To find the day of the week corresponding to any date, according to the Julian Calendar, or Old Style;

RULE.—To the given year add its one-fourth part (discarding fractions, if any,)—in leap years add its one-fourth part less one—the number of days that have elapsed since the beginning of the given year, and 5. The excess of sevens signifies the day of the week—1 indicating Sunday, 2 Monday, etc., while a remainder of 0 indicates Saturday.

The execution of Charles I occurred January 30, 1649; required the day of the week.*

Operation.

```
   1649 = Year.
    412 = Leap years.
     30 = Days since beginning of year.
      5
  -----
7)2096
  -----
    299 - 3 = Tuesday.
```

Columbus discovered America Oct. 12, 1492; required the day of the week.

Operation.

```
   1492 = Years.
    372 = Leap years, less one.
    286 = Days since beginning of year.
      5
  -----
7)2155
  -----
   307 - 6 = Friday.
```

EXPLANATION.—When the year is exactly divisible by 4 it is always a leap year, Old Style. 1492 being a leap year year we must be careful and count the extra day for February; also to deduct 1 from the leap years. In this case we have 286 days from the beginning of the year to the 12th of October.

*This being an English date it is of course counted O. S,, as it happened prior to 1752

NEW STYLE.

In devising a rule for telling the day of the week of any date according to New Style—which is our present method of reckoning time—we will first present a rule for the present century which is quite easy, and as there are no numbers to remember it should be known by every one, as it will not only answer as a calendar for daily use when no printed calendar is at hand, but all dates past and future can be easily ascertained. Corrections can be made from it for any century.

* *RULE.—To the given year add its one-fourth part (discarding fractions, if any)—in leap year add its one-fourth part* **less one**—*and the number of days that have elapsed since the beginning of the given year, inclusive. Divide their sum by 7. The excess of sevens signifies the day of the week. 1 signifies Sunday, 2 Monday, etc. An excess of 0 indicates Saturday.*

The battle of New Orleans was fought Jan. 8, 1815; find the day of the week.

Operation.

$$\begin{array}{r l}
1815 & = \text{Years.} \\
453 & = \text{Leap years.} \\
8 & = \text{Days elapsed since beginning of year.} \\
\hline
7\,)\,2276 & \\
\hline
325 - 1 & = \text{Sunday.}
\end{array}$$

Find the day of the week of July 4, 1876.

* In leap year do not forget the 29th of February when adding the number of days that have elapsed since the beginning of the year. Except in the case of the centurial years before mentioned leap year can always be told after dividing the year by 4, when the quotient is a whole number. Remember to deduct *one* in such years.

Operation.

$$
\begin{array}{rl}
1876 = & \text{Years.} \\
468 = & \text{Leap years } \textit{less one.} \\
31 = & \text{Days in January.} \\
29 = & \text{Days in February.} \\
31 = & \text{Days in March.} \\
30 = & \text{Days in April.} \\
31 = & \text{Days in May.} \\
30 = & \text{Days in June.} \\
4 = & \text{Days in July.} \\
\hline
7)\overline{2530} & \\
\hline
361 - 3 = & \text{Tuesday.}
\end{array}
$$

Explanation.—If the student will remember that for dates in the past century add 2, and for the next century add 5 to the operation before dividing by 7 the dates for 300 years can be easily ascertained. Thus, suppose we wish to find on what day of the week occurred the signing of the declaration of independence July 4, 1776. We proceed as above, with the date in 1876 and add 2 to the sum, which gives 2532. $2532 \div 7 = 361$, and a remainder of 5, which signifies Thursday, answer. In like manner, for the same date in 1976 we add 5 and get 2535, which divided by 7, as before, gives an exceess of 1, which signifies that July 4, 1976, will fall on Sunday.

The rule for telling the number of days to add for any century, New Style, is as follows: Find the figure to add for dates between 2000 and the century following. We reject the first two ciphers and we have 20. From this subtract 16 and multiply the remainder (4) by $5\frac{1}{4}$ (rejecting fractions) we get 21. We now add 4 to this result, which gives 25. Divide by 7, and we have 3 and a remainder of 4. The remainder is the figure sought.

A rule which applies to the present century and by making corrections as in the preceding rule to all centuries—New Style—can be applied mentally. To do this, figures which we shall term "excess figures" of the months must be memorized. They are as follows:

0	1	2	3	4	5	6
June.	Sept. Dec.	April July	Jan.* Oct.	May	Aug.	Feb.* Nov. March.

* January and February are each called **one less** in leap years.

Find the day of the week of Dec. 25, 1858.

$$
\begin{array}{rl}
8\,)\,\underline{5\,8\,0} & = \text{Last two figures of year with cipher annexed.} \\
7\,2 & = \tfrac{1}{8} \text{ Discarding fraction.} \\
2\,5 & = \text{Date of month.} \\
\underline{1} & = \text{Excess figure of December.} \\
7\,)\ \underline{9\,8} & \\
1\,4 - 0 & = \text{Saturday.}
\end{array}
$$

Great rapidity can be attained by rejecting the sevens as they occur. For instance, after dividing by 8 we have 72. 7 is contained in 72 10 times and a remainder of 2. Simply retain the remainder. Adding this to 25 we get 27, and discarding the sevens we have 6, which added to the excess figure (1) of December makes 7. As this is an even number of sevens we call the date Saturday, as previously explained.

MOON'S AGE.

Table of Monthly Numbers.

January	0	May	3	September	8
February	2	June	4	October	8
March	1	July	5	November	10
April	2	August	6	December	10

Find the age of the moon on the 20th of March, 1876.

Explanation.—To the given year we add 1 which gives 1877, and divide by 19, we have 98 and a remainder of 15. Retain the remainder unless it happens to be 0, when we must call it 19. We now multiply our remainder (15) by 11 which gives 165. From this substract the number of days suppressed by the reformation of the Calendar, which is 10 days if the year is between 1582 and 1700, 11 days if between 1700 and 1900, 12 days if between 1900 and 2200. According to this the number of days to substract from 165 is 11, which gives 154. We now divide 154 by 30, which gives 5 and a remainder of 4, which (4) added to the date of the month (20) we have 24. We also add the monthly number (1) which is found in the table and we have 25, which is the age of the moon on March 20 in 1876. If this last remainder be 30 it shows that the new moon took place on that day; but if it exceeds 30 substract 30 from it

and the remainder will be the age of the moon. In calculations of this nature great exactnesss must not be expected. The irregular arrangements of the months, the mean numbers necessary to be assumed in the formation of the periods from which these calculations are deduced and the irregularity of the lunar revolution may occasion an error of nearly 48 hours.

EASTER SUNDAY.

Easter Sunday should be celebrated on the first Sunday after the 14th day of the moon, if this 14th should happen on or after the 21st of March; therefore, Easter cannot happen earlier than March 22 nor later than April 25. The following table has been computed for easy reference:

EASTER TABLE.

Showing Easter Sunday for a period of 100 years, from which can be reckoned the dates of the celebration of Mardi Gras and all Church Feasts depending on Easter.

Septuagesima Sunday is 9 weeks, First Sunday in Lent is 6 weeks, Ash Wednesday is 46 days, and Mardi Gras is 47 days *before* Easter.

Rogation Sunday is 5 weeks, Ascension Day or Holy Thursday is 39 days, Pentecost or Whit Sunday is 7 weeks, and Trinity Sunday is 8 weeks *after* Easter.

1800...April 13	1826...March 26	1852...April 11	1878...April 21
1801...April 5	1827...April 15	1853...March 27	1879...April 13
1802...April 18	1828...April 6	1854...April 16	1880...March 28
1803...April 10	1829...April 19	1855...April 8	1881...April 17
1804...April 1	1830...April 11	1856...March 23	1882...April 9
1805...April 14	1831...April 3	1857...April 12	1883...March 25
1806...April 6	1832...April 22	1858...April 4	1884...April 13
1807...March 29	1833...April 7	1859...April 24	1885...April 5
1808...April 17	1834...March 30	1860...April 8	1886...April 25
1809...April 2	1835...April 19	1861...March 31	1887...April 10
1810...April 22	1836...April 3	1862...April 20	1888...April 1
1811...April 14	1837...March 26	1863...April 5	1889...April 21
1812...April 29	1838...April 15	1864...March 27	1890...April 6
1813...April 18	1839...March 31	1865...April 16	1891...March 29
1814...April 10	1840...April 19	1866...April 1	1892...April 17
1815...March 26	1841...April 11	1867...April 21	1893...April 2
1816...April 14	1842...March 27	1868...April 12	1894...March 25
1817...April 6	1843...April 16	1869...March 28	1895...April 14
1818...March 22	1844...April 7	1870...April 17	1896...April 5
1819...April 11	1845...March 23	1871...April 9	1897...April 18
1820...April 2	1846...April 12	1872...March 31	1898...April 10
1821...April 22	1847...April 4	1873...April 13	1899...April 2
1822...April 7	1848...April 23	1874...April 5	1900...April 15
1823...March 30	1849...April 8	1875...March 28	1901...April 7
1824...April 18	1850...March 31	1876...April 16	1902...March 3[illegible]
1825...April 3	1851...April 20	1877...April 1	1903...April 1[illegible]

MENSURATION

— OR —

Practical Geometry.

These pages present in simple form and language a ready reference for civil engineers, mechanics, farmers, teachers, students, and all who have occasion to refer to this important branch of mathematics. Many of the rules are entirely original, and the entire collection cannot fail to meet all requirements in measuring surfaces and solids that are met with in practical work.

SQUARES AND RECTANGLES.

GIVEN	TO FIND
Length and **Breadth.**	**Area** of any **Square** or **Rectangle.**

RULE.—*Multiply* the *length* by the *breadth.*

PROBLEM.—A field 10 rods long and 5 rods wide contains how many square rods?

Operation.—$10 \times 5 = 50$ sq. rods, area.

Remark.—Area of a *square* = one-half the square of its *diagonal.*

GIVEN	TO FIND
Area and **one** side of a **Square** or **Rectangle.**	The **Unknown Side** of any **Square** or **Rectangle.**

RULE.—*Divide* the *area* by the *given side.*

A field 10 rods long and containing 50 square rods, what is the width.

Operation.—$50 \div 10 = 5$ rods, width.

GIVEN	TO FIND
Side of a **Square.**	**Diagonal** of a given **Square.**

RULE.—*Multiply* the *length* of the given *Side* by 1.41421; or, *multiply* the *length* of the given *side* by 99 and *divide* the product by 70.

The side of a given square is 4 feet. Find the diagonal.

Operation.—1.41421×4 (length of side)=5.65684, or 99×4=396. 396÷70 =5.657, answer.

Remark.—The second rule gives slightly too much, but the error is of little consequence.

GIVEN	TO FIND
Diagonal of a **Square.**	The **Side** of a given **Square.**

RULE.—*Multiply* the *diagonal* of the *square* by .707107; or, *multiply* the *diagonal* by 70 and *divide* by 99.

The diagonal of a given square is 7 feet. Find the length of a side.

Operation.—.707107×7 (diagonal)—4.949749 feet, length of side.

Remark.—When figures are in the form of a rhombus, rhomboid, trapezium, or trapezoid care should be used in ascertaining the *true width.* Frequently a *side* of such a figure is mistaken for the width.

GIVEN	TO FIND
Area or **Diagonal** of a **Square**	**Area** of the **Circumscribed Circle** of a given **Square.**

RULE.—*Double* the *area* and *multiply* the *product* by .7854; or, *multiply* the *square* of the *diagonal* by .7854.

The area of a given square is 8 feet. Find the area of its circumscribed circle.

Operation.—8 (area)×2=16. .7854×16=12.5664 sq. ft., answer.

Diagonal of a given square is 4 feet. Find the area of its ciroumscribed.

Operation.—4^2 (diagonal)=16. .7854×16=12.5664 sq. ft., answer.

Remark.—It should be remembered that the *area* of an inscribed circle of a square is one-half the area of its circumscribed circle, and the same proportion exists between inscribed and described squares of circles.

GIVEN	TO FIND
Area or **Diagonal** of a **Square**	**Area** of the **Inscribed Circle** of a given **Square.**

Rule.—*Multiply* the *area* by .7854, or take one-half the *square* of the *diagonal* and *multiply* by .7854.

The area of a given square is 8 feet. Find the area of its inscribed circle.

Operation.—.7854×8=6.2832 sq. ft. area of inscribed circle.

Diagonal of a given square is 4 feet. Find the area of its inscribed circle.

Operation.—4^2 (diagonal)=16. 16÷2=8. .7854×8=6.2832 sq. ft. area of inscribed circle.

GIVEN	TO FIND
One Side or **Diagonal** of a **Square.**	**Radius** of the **Circumscribed Circle** of a **Square.**

Rule.—*Multiply* the given *side* by .707107; or, take *one-half* the *diagonal.*

The side of a given square is 5 feet. Find the radius of its circumscribed circle.

Operation.—.707107×5(side)=3.535535 ft., answer.

GIVEN	TO FIND.
Side of a **Square.**	**Diameter** of a **Circle** of **Equal Area.**

Rule.—*Multiply* the given *side* by 1.12838.

The side of a given square is 3 feet. Find the diameter of a circle containing an equal area.

Operation.—1.12838×3 (length of side)=3.38514 ft., answer.

GIVEN	TO FIND
Side of a **Square.**	The **Side** of an **Equilateral Triangle** of **Equal Area.**

Rule.—*Multiply* the *given side* by 1.51967.

The side of a given square is 3 feet. Find the side of an equilateral triangle which will contain an equal area.

Operation.—1.51967×3 (side of given square)=4.55901 ft., answer.

GIVEN	TO FIND
Length of **One Side** of a **Regular Polygon.**	**Area** of a **Regular Polygon.**

RULE.—*Square one side* of the *polygon* and for the

Equilateral triangle	(3 sides) ×	.433013
Pentagon	(5 sides) ×	1.720477
Hexagon	(6 sides) ×	2.598076
Heptagon	(7 sides) ×	3.633912
Octagon	(8 sides) ×	4.828427
Nonagon	(9 sides) ×	6.181824
Decagon	(10 sides) ×	7.694209
Undecagon	(11 sides) ×	9.365640
Dodecagon	(12 sides) ×	11.196152

Or, divide the *polygon* into *triangles* and find the *sum* of their *areas*; or, multiply *one-half* the *perimeter* by the *apothem.*

TRIANGLES.

GIVEN	TO FIND
Base and **Altitude**, or **Three Sides**, or the **Sum** of **Sides.**	**Area** of **any Triangle.**

RULE.—*Multiply base* by *half* the *altitude;* or, from the *half sum* of the *three sides substract* each sum separately, multiply the *half sum* and the *three remainders* continuously together and extract the *square root* of the *product.*

Sides of a triangle are 4, 7, and 9 feet, find area.

Operation.—4+7+9=20, sum of sides. 20÷2=10, half sum of sides. 10−4=**6.** 10−7=**3.** 10−9=**1.** **10×6×3×1**=180. Square root of 180=13.4+ square feet area.

GIVEN.	TO FIND
Area and **Altitude** of a **Triangle.**	The **Base** of any **Triangle.**

RULE.—*Divide twice* the *area* by the *altitude.*

The area of a given triangle is 6 feet, altitude 3 feet. Find base.

Operation.—6 (area)×2=12. 12÷3 (altitude)=4 ft., length of base.

GIVEN	TO FIND
Area and **Base** of a **Triangle.**	**Altitude** of any **Triangle.**

RULE.—*Divide twice* the *area* by the *base.*

The area of a given triangle is 6 feet, base 4 feet. Find altitude.
Operation.—6 (area)×2=12. 12÷4 (base)=3 ft., altitude.

GIVEN	TO FIND
The **Three Sides** of any **Triangle** or their **product.**	**Diameter** of the **Circumscribed Circle** of any **Triangle.**

RULE.—*Divide* the *product* of the *sides* of the triangle by *twice* the *area.*

The sides of a given triangle are 3, 4, and 5 feet. Find the diameter of its circumscribed circle.

Operation.—3×4×5=60, product of the sides. The area, found by a preceding rule is 6 ft. Twice the area =12 ft. 60÷12=5 ft., diameter of circumscribed circle.

GIVEN	TO FIND
Side of an **Equilateral Triangle.**	**Area** of an **Equilateral Triangle.**

RULE.—Multiply the *square* of the *given side* by .433013; or, multiply the *apothem* by *half* the *perimeter* (sum of sides).

One side of an equilateral triangle is 4 feet. Find the area.
Operation.—4^2 (side)=16. .433013×16=6.928208 sq. ft., area.

GIVEN	TO FIND
Side of an **Equilateral Triangle.**	**Altitude** of an **Equilateral Triangle.**

RULE.—Multiply the *given side* by .866025.

The side of a given equilateral triangle is 4 feet. Find the altitude.
Operation.—.866025×4 (side)=3.464100 ft., altitude.

GIVEN	TO FIND
Altitude of an **Equilateral triangle.**	**Side** of an **Equilateral triangle.**

RULE.—*Multiply* the *altitude* by 1.1547, or *divide* the *altitude* by .866025, or *multiply* the *square root* of the *area* by 1.51967.

GIVEN	TO FIND
Side of an **Equilateral Triangle.**	**Side** of a **Square** of **equal area.**

RULE.—*Multiply* the *given side* by .658037.

The side of a given equilateral triangle is 6 feet. Find the length of one side of a square which will contain the same area.

Operation.—$.658037 \times 6 = 3.948222$ ft., answer.

GIVEN	TO FIND
Side of an **Equilateral Triangle.**	**Diameter** of a **Circle** of **equal area.**

RULE.—*Multiply* the given *side* of the *triangle* by .742517.

What is the diameter of a circle which will contain the same area as an equilateral triangle whose sides are 6 feet each.

Operation.—$.742517 \times 6$ (side) $= 4.455102$ ft., diameter.

GIVEN	TO FIND
Side of an **Equilateral Triangle.**	**Diameter** of the **Circumscribed Circle** of an **Equilateral Triangle.**

RULE.—*Multiply* the *length* of the *given side* by 1.1547.

The side of an equilateral triangle is 4 feet. Find the diameter of its circumscribed circle.

Operation.—$1.1547 \times 4 = 4.6188$ ft. diameter.

GIVEN	TO FIND
Altitude of an **Equilateral Triangle.**	**Apothem or Diameter** of the **Inscribed Circle** of an **Equilateral Triangle.**

RULE.—Take *one-third* of the *altitude.*

Altitude of an equilateral triangle is 6 feet. Find the diameter of its inscribed circle.

Operation.—6 (diameter) ÷3=2 ft.. answer.

Remark.—One-fourth of the *diameter* of its *circumscribed circle* of an equilateral triangle also equals the *diameter* of its *inscribed circle* or apothem.

GIVEN	TO FIND
Base and **Altitude** of a **Right** angled **Triangle.**	Hypotenuse of a **Right Angled Triangle.**

RULE.—Extract the *square root* of *sum* of the *squares* of the *base* and *altitude.*

Base and altitude of a given right angled triangle are 5 feet and 12 feet respectively. Find the hyptenuse.

Operation.—$5^2+12^2=169$. Square root of 169=13 ft., hypotenuse.

GIVEN	TO FIND
Base and **Hypotenuse** of a **Right Angled Triangle.**	**Altitude** of a **Right Angled Triangle.**

RULE.—From the *square* of the *hypotenuse substract* the *square* of the *base* and extract the *square root* of the remainder.

The hypotenuse and base of a right angled triangle are 13 feet and 5 feet, respectively. Find the altitude.

Operation.—$13^2=169$. $5^2=25$. $169-25=144$. Square root of 144=12 ft., altitude.

GIVEN	TO FIND
Altitude and **Hypotenuse** of a **Right Angled Triangle.**	**Base** of a **Right Angled Triangle.**

RULE.—From the *square* of the *hypotenuse substract* the *square* of the *altitude* and extract the *square root* of the remainder.

The hypotenuse and altitude of a right angled triangle are 13 feet and 12 feet respectively. Find the base.

Operation.—$13^2=169$. $12^2=144$, $169-144=25$. Square root of 25=5 ft., base.

THE CIRCLE.

GIVEN	TO FIND
Diameter or **Area** of a **Circle.**	**Circumference** of a **Circle.**

RULE.—*Multiply* the *diameter* by $3\frac{1}{7}$, or 3.1416, or for greater "accuracy" by 3.141592653, or *multiply* the *area* by 12.566 and extract the *square root* of the *product.*

Remark.—Archimedes found that the circumference of any circle was *nearly* $3\frac{1}{7}$ times the diameter. If the *exact* ratio could be found the process of squaring or quadraturing the circle would be accomplished by figures. Thousands of persons have claimed in almost as many different ways the accomplishment of this *impossibility.* The ratio of the diameter to the circumference has been found to over 600 decimal places. The preceding ratios will answer all practical purposes, but we will append the following, which gives the ratio of the diameter to the circumference to be as

1 to 3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679821480865132823066470938 + to infinity.

GIVEN	TO FIND.
Circumference or **Area** of a **Circle.**	**Diameter** of a **Circle.**

RULE.—*Multiply* the *circumference* by .31831, or *multiply* the *area* by 1.2732 and extract the *square root* of the *product.*

The circumference of a given circle is 9 feet. Find the diameter.
Operation.—.31831×9 (circumference) = 2.86479 ft., diameter.

GIVEN.	TO FIND
Diameter or **Circumference** of a **Circle.**	**Area** of a given **Circle.**

RULE.—*Square* the *diameter* and *multiply* by .7854, or for greater "accuracy" *multiply* by .78539816, or *square* the *circumference* and *multiply* by .07957747, or *multiply half* the *diameter* by *half* the *circumference*, or *square* the *radius* and *multiply* by 3.1416.

GIVEN	TO FIND
Diameter or **Circumference** of a **Circle.**	**Side** of the **Inscribed Square** of a **Circle.**

RULE.—*Multiply* the *diameter* by .707107, or *multiply* the *circumference* by .22508.

The diameter of a given circle is 8 feet. Find the side of the largest square that can be inscribed therein.

Operation.—.707107×8(diameter)=5.656856 ft., answer.

GIVEN	TO FIND
Diameter of a **Circle.**	**Area** of the **Inscribed Square** of a **Circle.**

RULE.—Take *one-half* the *square* of the *diameter.*

GIVEN	TO FIND
Diameter of **Circle.**	**Area** of the **Described Square** of a **Circle.**

RULE.—*Square* the *diameter.*

GIVEN	TO FIND
Diameter of a **Circle.**	**Side** of an **Equilateral Triangle** of **equal Area.**

RULE.—*Multiply* the *diameter* by 1.3468.

CIRCULAR ARCS AND CHORDS.

GIVEN	TO FIND
Length of **Arc** and **Circumference** of a **Circle.**	**Number** of **Degrees** in a **Circular Arc.**

RULE.—*Multiply* the *length* of the given *arc* by 360 and *divide* by the *circumference.*

The circumference of a given circle is 50 feet, and the length of the given arc is 8 feet. Find the number of degrees in the arc.

Operation.—$360 \times 8 = 2880$. $2880 \div 50 = 57\frac{3}{5}$, answer.

GIVEN	TO FIND
The **Degrees** of an **Arc** of a **Circle.**	**Length** of a **Circular Arc.**

RULE.—*Multiply* the number of *degrees* by the circumference and *divide* by 360, or *multiply* the *chord* of *half* the *arc* by 8, *substract* the *chord* of the *whole arc* and divide the remainder by 3, or *multiply* the number of *degrees* in the arc by the *diameter* of the *circle* and the product by .0087266.

GIVEN	TO FIND
Height of **Arc** and **Chord** of **half** the **Arc.**	**Diameter** of the **Circle** of a given **Arc.**

RULE.—*Divide* the *square* of the *chord* of *half* the *arc* by the *height* of the *arc.*

GIVEN	TO FIND
Chord of **half** the **Arc** and **Diameter** of the **Circle.**	**Height** of a **Circular Arc.**

RULE.—*Divide* the *square* of the *chord* of *half* the *arc* by the *diameter* of the *circle.*

The chord of half an arc is 12 feet and the diameter of the circle is 36 feet. Find the height of the arc.

Operation.—$12^2 = 144$. $144 \times 36 = 4$ ft., height of arc.

GIVEN	TO FIND
Height of **Arc** and **Diameter** of the **Circle.**	The **Chord** of **Half** the **Arc.**

RULE.—*Multiply* the *diameter* of the circle by the *height* of the *arc* and extract the *square root* of the *product.*

The height of an arc is 4 feet, and the diameter of the circle is 36 feet. Find the length of the chord of half the arc.

Operation.—$36 \times 4 = 144$. Square root of $144 = 12$ ft., length of chord of half the arc.

GIVEN	TO FIND
Length of **Chord** and **Height** of an **Arc.**	**Diameter** of a **Circle** of a **Circular Arc.**

RULE.—*Divide* the *square* of *half* the *chord* by the *height* and to the *quotient add* the given height.

The length of a given chord is 56 feet and the height 8 feet. Find the diameter of the circle.

Operation.—56 (length of chord) ÷ 2 = 28. $28^2 = 784$. 784 ÷ 8 (height) = 98. 98 + 8 = 106 ft., diameter of the circle.

GIVEN	TO FIND
The **Height** of an **Arc** and **Diameter** of the **Circle.**	**Length** of the **Chord** of a **Circular Arc.**

RULE.—Multiply the *difference* of the *height* of the *arc* and the *diameter* of the circle by the *height* of the *arc*; extract the *square root* of the result and multiply by 2.

The height of an arc is 2 feet and the diameter of the circle is 10 feet. Find the length of the chord of the arc.

Operation.—10 (diameter) − 2 (height of arc) = 8. 8×2 (height of arc) = 16. Square root of 16 = 4. 4×2 = 8 ft., chord of arc.

GIVEN	TO FIND
The **Length** of the **Chord** of an **Arc** and **Diameter** of the **Circle.**	**Height** of a **Circular Arc.**

RULE.—Take *half* the *diameter* of the *circle* and half the *length* of the *chord* of the *arc.* Multiply their *sum* by their difference, extract the *square root* of the product, and subtract the square root thus found from *half* the *diameter* of the *circle.*

The chord of an arc is 8 feet and the diameter of the circle is 10 feet. Find the height of the arc.

Operation.—4 = ½ diameter. 5 = ½ length of arc. 4 + 5 = 9, sum of half the diameter and half the length of the arc. 5 − 4 = difference of half the diameter and half the length of the arc. 9×1 = product of difference and sum. Square root of 9 = 3. 5 (half diameter) − 3 (sq. rt.) = 2 ft., height of arc.

GIVEN	TO FIND
Length of **Chord** and **Diameter** of the **Circle.**	**Chord** of **Half** a **Circular Arc.**

RULE.—Find the *height* of the *arc* by the preceding rule, and to its *square add* the *square* of *half* the *length* of the *chord* of the *arc* and extract the square root of the sum.

The chord of an arc is 14 feet and the diameter of the circle is 50 feet. Find the length of the chord of half the given arc.

Operation.—The height of the arc, found by the preceding rule, is 1 ft. Half of the chord (14 ft.)=7. $7^2=49$, to which add the square of the height (1^2) 1, we get 50. Square root of 50=7.071+ft., length of cord of half the given arc.

GIVEN	TO FIND
Chord of **Half** an **Arc** and **Diameter** of the **Circle.**	The **Chord** of a **Circular Arc.**

RULE.—Divide the *square* of the *chord* of *half* the *arc* by the *diameter* of the *circle*, subtract the *square* of the *quotient* from the *square* of the given *length* of the *chord* of *half* the *arc*, and extract the *square root*. Then multiply the root thus found by 2.

The chord of half an arc is 12 feet and the diameter of the circle is 36 feet. Find the chord of the given arc.

Operation.—12^2 (length of chord of half the arc)=144. 144÷36 (diameter of circle)=4. $4^2=16$. 12^2 (chord of half the arc)=144. 144−16=128. Square root of 128=11.31+. 2×11.31=22.62+ft., length of chord of arc.

GIVEN	TO FIND
Chord of an **Arc** and **Chord** of **Half** the **Arc.**	The **Diameter** of the **Circle** of a **Circular Arc.**

RULE.—From the *square* of the *chord* of *half* the arc subtract the *square* of *half* the *chord* of the *whole* arc and extract the *square root* of the remainder. Divide the *square* of the *chord* of *half* the arc by the *root* thus found.

The chord of an arc is 48 feet and the chord of half the arc is 26 feet. Find the diameter of the circle.

Operation.—48 (chord of whole arc)÷2=24. $24^2=576$. 26^2 (chord of half the arc)=676. 676−576=100. Square root of 100=10. 676 (square of chord of half the arc)÷10=67.6 ft., diameter of circle.

GIVEN	TO FIND
Length of **Arc** and **Radius** of **Circle**, or **Area** of **Circle** and **Degrees** of **Angle** of **Sector.**	**Area** of a **Sector** of a **Circle.**

RULE.—Multiply the *length* of the *arc* by the *radius* and take half the *product;* or, multiply the *area* of the *circle* by the number of *degrees* in the *angle* of the *sector* and divide by 360.

PROBLEM.—The radius of a circle is 10 feet and the angle of the sector 40 degrees. Find the area of the sector.

Operation.—First find area of circle. 10 (radius)$\times$2=20 diameter. 20^2=400. .7854$\times$400=314.1600 sq. ft., area of circle. 314.16 (area)$\times$40 (No. degrees in angle of sector)=12566.40, which divided by 360=34.90+sq. ft., area of sector.

PROBLEM 2.—(Length of arc given.) The length of the arc is 6 feet and the radius 8 feet. Find the area of the sector.

Operation.—8$\times$6=48. 48$\div$2=24 sq. ft., area of sector.

GIVEN	TO FIND
Angle and **Radius** of a **Circle.**	**Area** of a **Segment** of a **Circle.**

RULE.—Find the *area* of the *sector* which has the same *arc* and substract the *area* of the *triangle* formed by the radii and the chord.

The radius of a circle is 10 feet, and the angle of the sector is 60 degrees. Find the area of the segment.

Operation—10$\times$10$\times$3.1416=314.16, area of circle. As the area of the sector is 60 degrees or $\frac{1}{6}$ of the circle, its area is $\frac{1}{6}$ of 314.16 or 52.36 sq. ft. The triangle in this case is equilateral and the area 43.30ft. subtracted from 52.36=9.06 sq. ft. area of segment.

GIVEN	TO FIND
Length of **Chord** and **Height** of **Arc** of a **Circle.**	**Area** of a **Segment** of a **Circle.**

RULE.—*Add* together *one-fourth* of the *square* of the *chord* and *two-fifths* of the *square* of the *height* and multiply the *square root* of the *sum* by $\frac{4}{3}$ of the *height.*

This rule taken from Todhunter's Mensuration is not exact, as it gives the area of the segment greater than it ought to be, but the error is very small provided the angle of the corresponding sector is small; when this area is 60 degrees, the error is less than $\frac{1}{20000}$ part of the area, and when this angle is 90 degrees the area is less than $\frac{1}{4000}$ part of the area.

THE ELLIPSE.

GIVEN	TO FIND
Both Diameters.	**Area.**

RULE.—Multiply the *product* of *both diameters* by .7854.

RECTANGULAR SOLIDS.

GIVEN	TO FIND
Length, Width, and **Height.**	**Cubic Contents** of a **Cube** or any **Rectangular Solid.**

RULE.—Multiply together the *length*, *width*, and *height*.

A piece of timber 10 inches long, 6 inches high, and 4 inches wide. Find the cubic contents.

Operation.—$10 \times 6 \times 4 = 240$ cu. in., solidity.

Remark.—To change cubic inches to cubic feet divide cubic inches by 1728.

GIVEN	TO FIND
Cubic Contents and **Lengths** of **Two Sides.**	**Length** of an **Unknown Side** of a **Rectangular Solid.**

RULE.—Divide the *cubic contents* by the product of the *two given sides.*

What should be the length of a box necessary to contain 240 cubic inches whose height is 6 inches and width 4 inches?

Operation.—4 (width) $\times$ 6 (height) $= 24$. $240 \div 24 = 10$ in., length.

GIVEN	TO FIND
Length of a **Side** of a **Cube.**	**Diagonal** of a **Cube** or **Diameter** of its **Circumscribed Sphere.**

RULE.—Multiply the *length* of the *side* by 1.7320508.

PRISMS.

GIVEN	TO FIND
Length of **Side** and **Height** of a **Prism.**	**Cubic Contents** of a **Prism.**

RULE.—Multiply the *area* of the *base* by the *height.*

Remark.—The base of a prism is usually one of the regular polygons The rules for finding their areas are on page 69.

Find cubic contents of a triangular prism, the length of one side of the base being 4 feet and the height 8 feet.

Operation.—We find the area of the base by rule on page 69. Thus, $4^2=16$. $.433013\times16=6.928208$, sq. ft. area of base, which multiplied by 8 (height) given 55.425664 cu. ft., answer.

GIVEN	TO FIND
Cubic Contents and **Area** of the **Base** of a **Prism.**	The **Height** of a **Prism.**

RULE.—Divide the *cubic contents* by the *area* of the *base.*

Find the height of a hexagonal prism necessary to contain 40 cubic feet. The length of each side is 9 feet.

Operation.—We find the area of the base, as in the preceding example, to be 23.31684 sq. ft., which divided by 40 (desired contents)=1.7154 ft. high, answer.

Remark.—The decimal of a linear foot can be reduced to inches by multiplying it by 12. Thus, $.7154\times12=8.5848$ linear inches, *i. e.*, a little more than 8½ inches.

GIVEN	TO FIND
Perimeter and **Height** of a **Prism.**	**Area** of the **Lateral Surface** of a **Prism.**

RULE.—Multiply the *perimeter* by the *height.*

The length of a side of a pentagonal prism (5 sided) is 6 inches and its height 20 inches. Find the area of its lateral surface.

Operation.—5 (number of sides)$\times$6 (length of one side)=30 (perimeter, or sum of all sides). $30\times20=600$ sq. in., area.

Remark.—Lateral surface means surface of the sides. If the area of the *whole* surface is required, add to the lateral surface the areas of the ends.

GIVEN	TO FIND
Area of the **Base** and **Height** of a **Prism.**	**Cubic Contents** of a **Prism.**

RULE.—Multiply the *area* of the *base* by the *height* and take *one-third* of the product.

The side of a quadrangular prism (4 sided) is 6 feet and the height 10 feet. Find the cubic contents.

Operation.—$6^2=36$ sq. ft., area of base. 36×10 (height) $=360$. $360\div3=120$ cu. ft., contents.

Remark.—It will thus be seen that the contents of any pyramid is *one-third* of the contents of a *prism* of equal base and height.

PYRAMIDS.

GIVEN	TO FIND
The **Slant Height** and **Perimeter** of a **Pyramid.**	The **Area** of the **Lateral Surface** of a **Pyramid.**

RULE.—Multiply the *perimeter* of the *base* by the *slant height* and take *one-half* the *product.*

Find the lateral surface of a triangular pyramid the perimeter of which is 27 inches and slant height 18 inches.

Operation.—18 (slant height) $\times27$ (perimeter) $=486$. $486\div2=243$ sq. in., area of lateral surface.

Remark.—It should be known that pyramids, as well as cones, have both a *true* and *slant* height. The true height is the *shortest distance* from the *apex* to the *base.* The *slant height* is the *shortest distance* from the *apex* to the *perimeter.*

GIVEN	TO FIND
Area of the **Base** and **Cubic Contents** of a **Pyramid.**	The **Height** of a **Pyramid.**

RULE.—Divide *three times* the *contents* by the *area* of the *base.*

Remark.—This rule also applies to cones.

The area of the base of a pyramid is 40 square inches and the contents is 300 cubic inches. Find the height.

Operation.—300 (cu. contents) $\times3=900$. $900\div40$ (area of base) $=22\frac{1}{2}$ in. high.

GIVEN	TO FIND
Cubic Contents and **Height** of a **Pyramid.**	The **Area** of the **Base** of a **Pyramid.**

RULE.—Divide the *contents* by the *height.*

GIVEN	TO FIND
Areas of the **Two Ends** and **Height** of a **Frustum** of a **Pyramid.**	The **Cubic Contents** of a **Frustum** of a **Pyramid.**

RULE.—To the *sums* of the *areas* of the *two ends* of the frustum *add* the square root of their *product.* Multiply the sum by the *height* and take *one-third* of the product.

Find the cubic contents of a frustum of a pyramid whose height is 15 feet. The area of one end is 18 square feet and the other 98 square feet.

Operation.—18+98=116 (area of the two ends). 98×18=1764 square root of 1764=42. 116+42=158. 15 (height)×158=2370, which divided by 3 gives 790 cu. ft., answer.

Remark.—This rule also applies to frustums of cones, but further on we give a rule which is much shorter for such cases.

GIVEN	TO FIND
Both Perimeters of a **Frustum** of a **Pyramid.**	The **Area** of the **Surface** of a **Frustum** of a **Pyramid.**

RULE.—Multiply *half* the *sum* of the *upper* and *lower perimeter* by the *slant height* of the frustum.

Each of the sides of a frustum of a hexagonal pyramid (6 sided) is 2 feet at top and 4 feet at the bottom. The height is 12 feet. Find the area of the surface.

Operation.—2 (width of one side at top)×6 (number of sides)=12, perimeter at top. 4 (width of one side at bottom)×6 (number of sides)= 24, perimeter at bottom. 24+12=36. 36÷2=18, mean perimeter. 18×12= 216 sq. ft., area.

Remark.—This gives the area of the sides, to which add the areas of the ends if required.

CYLINDERS.

GIVEN	TO FIND
Diameter and **Height** of a **Cylinder.**	**Cubic Contents** of a **Cylinder.**

RULE.—Multiply the *square* of the *diameter* by .7854 and the *product* by the *height.*

Find the cubic contents of a cylinder 3 feet in diameter and 7 feet high.

Operation.—3^2 (diameter)=9. .7854×9=7.0686, area of base. 7.0686×7 (height)=53.4802 cu. ft., answer.

GIVEN	TO FIND.
Diameter and **Height** of a **Cylinder.**	The **Cylindrical Contents** of a **Cylinder.**

RULE.—*Square* the *diameter* and multiply by the *height.*

GIVEN.	TO FIND
Diameter and **Cylindrical Contents** of a **Cylinder.**	**Height** of a **Cylinder.**

RULE.—Divide the *cylindrical contents* by the *square* of the *diameter.*

Find the height of a cylindrical can necessary to contain 3718 cylindrical inches.

Operation.—13^2=169. 3718÷169=22 inches, height.

Remark.—Cylindrical contents multiplied by .7854=cubic contents.

GIVEN	TO FIND
Height and **Cylindrical Contents** of a **Cylinder.**	**Diameter** of a **Cylinder.**

RULE.—Divide the *cylindrical contents* by the *height* and extract the square root of the *quotient.*

A cylindrical can 41 inches high and containing 8036 cylindrical inches. Find the diameter.

Operation.—8036 (cyl. contents)÷41 (height)=196, circular area of base. Square root of 196=14 inches diameter.

GIVEN	TO FIND
Circumference and **Height** of a **Cylinder.**	The **Curved Surface** of a **Cylinder.**

RULE.—Multiply the *circumference* of one of the *ends* by the *height.*

Remark.—Add to the curved surface the areas of both ends if the area of the entire surface is required.

CONES.

GIVEN	TO FIND.
Diameter of **Base** and **True Height** of a **Cone.**	The **Cubic Contents** of a **Cone.**

RULE.—Multiply the *square* of the *diameter* of the *base* by .7854 and the *product* by the *height* and divide by 3.

Remark.—It will be observed that the contents of a cone is *one-third* of a *cylinder* of *equal base* and *altitude.*

The diameter of a cone at the base is 4 feet and the height 6 feet. Find cubic contents.

Operation.—$4^2=16$. $.7854\times16=12.5664$ sq. ft., area of base. 12.5664×6 (height)$=75.3984$ cubic contents of a cylinder of equal base and altitude, which divide by $3=25.1328$ cubic contents of cone.

GIVEN	TO FIND
Diameter and **Slant Height** of a **Cone.**	**Area** of the **Curved Surface** of a **Cone.**

RULE.—Multiply the *circumference* of the *base* by the *slant height* and divide by 2.

Remark.—To which add the area of the base if the whole surface is required.

Find the curved surface of a cone whose slant height is 6 feet and diameter 2 feet.

Operation.—We first find circumference of base. 3.1416×2(diameter)$\approx$ 6.2832, which we multiply by the slant height (6), which gives 37.6992. We take one-half of the result and we have 18.8496 sq. ft., surface.

Remark 2.—If the diameter and *true height* are given we can find the *slant height* by squaring the true height and adding the product to the square of the radius and extract the square root of their sum.

GIVEN	TO FIND
Diameter and **Altitude** of a **Cone.**	**Cylindrical Contents** of a **Cone.**

RULE.—Multiply the *square* of the *diameter* by the *altitude* and divide the product by 2.

GIVEN	TO FIND
Diameter and **Altitude** of a **Cone.**	**Cubic Contents** of a **Cone.**

RULE.—Find the *cylindrical contents* by the preceding rule and multiply by .7854.

GIVEN	TO FIND
Cylindrical Contents and **Diameter** of a **Cone.**	The **Altitude** of a **Cone.**

RULE.—Multiply the *cylindrical contents* by 3 and divide the *product* by the *square* of the *diameter.*

The diameter of a cone is 4 feet and the cylindrical contents is 48 feet. Find the altitude or true height.

Operation.—Cyl. ft. $=48\times3=144$. $4^2=16$ square of diameter, $144\div16=9$ ft., altitude.

GIVEN	TO FIND
Altitude and **Cylindrical Contents** of a **Cone.**	**Diameter** of a **Cone.**

RULE.—Multiply the *cylindrical contents* by 3, divide the product by the *altitude* and extract the *square root* of the *quotient.*

The height of a cone is 12 feet and the cylindrical contents are 196 feet. Find the diameter.

Operation.—$196\times3=588$. $588\div12=49$. Square root of $49=7$ ft., diameter.

GIVEN.	TO FIND
The **Upper** and **Lower Diameters** and **Height** of a **Frustum** of a **Cone.**	**Cylindrical Contents** of a **Frustum** of a **Cone.**

RULE.—To the *sum* of the *squares* of the *upper* and *lower diameters* add the *product* of *both diameters.* Multiply the *complete sum* by the *height* and divide by 3.

Find the cylindrical contents of a tank in the form of a frustum of a cone whose upper and lower diameters are 3 and 5 feet respectively, and whose height is 7 feet.

Operation.—$3^2=9$ square of upper diameter. $5^2=25$ square of lower diameter. $3\times5=15$ product of both diameters. $9+25+15=49$. 7 (height) $\times49=343$. $343\div3=114\frac{1}{3}$ cyl. ft., contents.

GIVEN	TO FIND
Upper and **Lower Diameters** and **Height** of a **Frustum** of a **Cone.**	**Cubic Contents** of a **Frustum** of a **Cone.**

RULE.—Find the *cylindrical contents* by the preceding rule and *multiply* by .7854.

GIVEN	TO FIND
The **Two Diameters** and **Cylindrical Contents** of a **Frustum** of a **Cone.**	The **Height** of a **Frustum** of a **Cone.**

RULE.—Divide the *cylindrical contents* by *one-third* of the *product* of the *two diameters* and their *squares.*

A frustum of a cone whose upper diameter is 3 feet and lower diameter 6 feet contains 252 cylindrical feet. Find the height.

Operation.—Square of upper diameter=9. Square of lower diameter =36. 6×3=18, product of the two diameters. 9+36÷18=63. 63÷3=21. 252 (cyl. ft.)÷21=12 ft., height.

GIVEN	TO FIND
The **Two Diameters** and **Slant Height** of a **Frustum** of a **Cone.**	The **Curved Surface** of a **Frustum** of a **Cone.**

RULE.—Multiply *one-half* the *sum* of the *upper* and *lower circumferences* by the *slant height.*

THE GLOBE OR SPHERE.

GIVEN	TO FIND
The **Diameter** of a **Sphere.**	The **Cubic Contents** of a **Sphere.**

RULE.—*Cube* the *diameter* and multiply by .5236.

How many cubic feet are contained in a round ball or globe whose diameter is 5 feet?

Operation.—5×5×5=125. .5236×125=65.45 cu. ft., volume.

Remark.—The volume of a sphere also equals the cube of its circumference multiplied by .01689, or cube of radius multiplied by 4.1888, or its surface multiplied by one-sixth of diameter, or multiply the surface by diameter and divide by 6, or area of 4 circles of equal diameter multiplied by one-sixth of its diameter.

GIVEN	TO FIND
Diameter of a **Sphere.**	**Surface** of a **Sphere.**

RULE.—*Square* the *diameter* and *multiply* by 3.1416.

Remark.—The surface of a sphere also equals square of circumference multiplied by .31831, or the diameter multiplied by the circumference, or four times the area of a circle of equal diameter, or the square of the radius multiplied by 12.5664, or the solidity divided by one-sixth of its diameter.

GIVEN	TO FIND
Inner and **Outer Diameter** of a **Sperical Shell.**	**Cubic Contents** of a **Spherical Shell.**

RULE.—Subtract the *cube* of the *inner diameter* from the *cube* of the *outer diameter* and multiply the *result* by .5236.

The inner diameter of a spherical shell is 7 inches and the outer diameter is 9 inches. Find the cubic contents of the shell.

Operation.—The cube of 9 is 729. The cube of 7 is 343. $729-343=386$. $386\times.5236=202.1096$ cu. in., contents.

GIVEN	TO FIND
Radii of the **Two Ends** and **Height** of a **Spherical Zone.**	The **Cubic Contents** of a **Spherical Zone.**

RULE.—Add the *square* of the *height* to *three times* the *sum* of the *squares* of the *radii* of the *two ends*, and multiply the *complete sum* by the *height* and by .5236.

The radii of the ends of a spherical zone are 8 and 10 inches and the height is 3 inches. Find the cubic contents.

Operation.—$8^2=64$. $10^2=100$. $64+100=164$. $164\times3=492$. $492+9$ (square of height)$=501$. 501×3 (height)$=1503$. $1503\times.5236=786.9708$ cu. in., contents.

GIVEN	TO FIND
Height of **Spherical Zone** or or **Segment** and the **Circumference** of its **Sphere.**	The **Curved Surface** of a **Spherical Zone** or **Segment.**

RULE.—Multiply the *circumference* of the *sphere* by the *height* of the *zone* or *segment.*

The height of a segment of a sphere is 6 inches and the diameter of the sphere is 18 inches. Find the area of the curved surface.

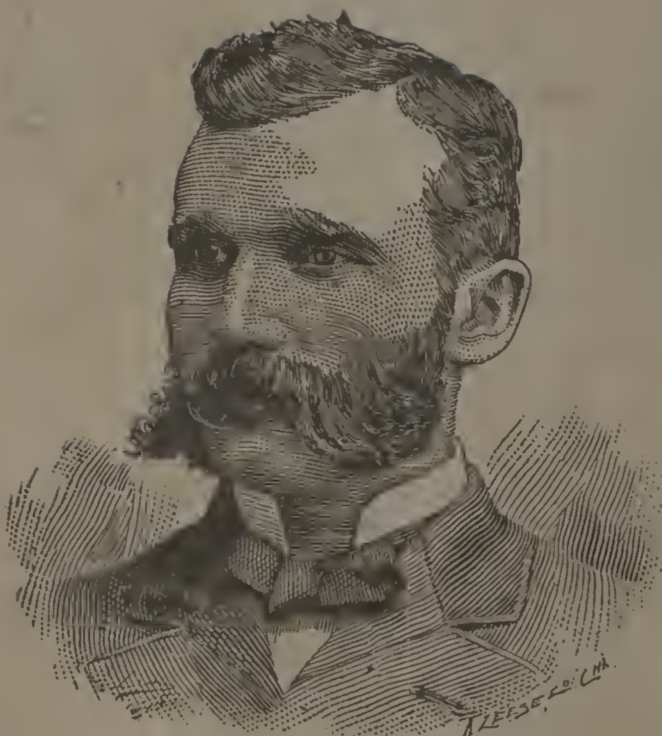
Operation.—We first find the circumference, thus: 3.1416×18 (diameter)$=56.5488$. 56.5488×6 (height)$=339.2928$ sq. in., surface.

GIVEN	TO FIND
Radius of the **Base** and **Height** of a **Spherical Segment**	The **Cubic Contents** of a **Spherical Segment.**

RULE.—To the *square* of the *height* add *three times* the *square* of the *radius* of the *base.* Multiply the *sum* by the *height* and by .5236.

The radius of the base of a spherical segment is 5 inches and the height is 3 inches. Find the cubic contents.

Operation.—5^2 (radius of base)=25. 25×3=75. 3^2=9 (square of height). 75+9=84. 84×3 (height)=252. 252×.5236=131.9472 cu. in.



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